

NAME \_\_\_\_\_ INDEX NUMBER \_\_\_\_\_

SCHOOL \_\_\_\_\_ DATE \_\_\_\_\_

## MATRICES AND TRANSFORMATIONS

<i>KCSE 1989 – 2012 Form 4 Mathematics</i> <i>Answer all the questions</i>	Working space
<p>1. <b>1989 Q12 P2</b></p> <p>The point (5, 2) undergoes the transformation <math>\begin{bmatrix} 3 &amp; 2 \\ -1 &amp; 0 \end{bmatrix}</math></p> <p><math>\begin{bmatrix} 3 &amp; 2 \\ -1 &amp; 0 \end{bmatrix}</math> followed by a translation <math>\begin{bmatrix} -6 \\ 11 \end{bmatrix}</math> .</p> <p>Determine the coordinates of the image. (3 marks)</p>	
<p>2. <b>2. 1990 Q16 P1</b></p> <p>The vertices of a triangle ABC are A(-1, -4), B (3,3) and C (2, 5). Find the image of the triangle under the transformation whose matrix is <math>\begin{pmatrix} 4 &amp; -2 \\ 6 &amp; -3 \end{pmatrix}</math></p> <p>Draw the triangle and its image on the same axis. (Grid was provided) (3 marks)</p>	

		Working space
3.	<p><b>1990 Q21 P2</b></p> <p>A parallelogram whose vertices are A (1, 0), B (3, 0), C (4, 2) and D (2, 2) is mapped onto a parallelogram A' B' C' D' by a transformation whose matrix is M. Under the transformation the images of A and C are A' (0, 1) and C' (-2, 4) respectively.</p> <p>i) Find matrix M (3 marks)</p> <p>ii) Plot the parallelogram ABCD on its image on the given grid (3 marks)</p> <p>iii) A' B' C' D' is mapped onto ABCD by a transformation T. Obtain the matrix for T. (2 marks)</p>	
4.	<p><b>1991 Q4 P1</b></p> <p>The image of a point A, under the transformation represented by the matrix</p> $T = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ <p>is A' (-2, 4). Find the coordinates of a. (3 marks)</p>	
5.	<p><b>1991 Q21 P2</b></p> <p>A rectangle OABC has the vertices O (0, 0), A (2,0), B (2, 3) and C (0, 3). O' A' B' C' is the image of OABC under the translation given by <math>\begin{bmatrix} 0 \\ 4 \end{bmatrix}</math>. O'' A'' B'' C'' is the image of O' A' B' C' under the transformation given by the matrix</p>	

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} .$$

Draw the rectangles OABC, O' A' B' C' and O'' A'' B'' C'' on the grid provided. (grid was provided).

(6 marks)

Use your diagram to find the centre of rotation which maps OABC onto O'' A'' B'' C''

(2 marks)

Working space

6. **1992 Q18 P1**

A point p ( $x', y'$ ) is the image of a point p( $x, y$ ) under a transformation, T, given by the matrix  $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

a) Express  $x'$  and  $y'$  in terms of  $x$  and  $y$

(3 marks)

b) If a point Q and its image Q' under the transformation T lie on the same line  $y=mx$ , Find two possible values of  $m$

(5 marks)

7. **1993 Q12 P1**

	<p>A transformation is represented by the matrix <math>\begin{bmatrix} 1 &amp; 2 \\ 3 &amp; 2 \end{bmatrix}</math>.</p> <p>This transformation maps a triangle ABC of the area <math>3\text{cm}^2</math> onto another triangle A'B'C'. Find the area of triangle A'B'C'.</p>	<p>Working space</p>
<p>8.</p>	<p><b>3. 1993 Q19 P1</b></p> <p>A transformation <math>T_1</math> maps the triangle ABC whose coordinates are A (-2, 0), B(1, -2) C (0, 1) onto triangle ABC whose coordinates A'(2, 4), B'(4,1), and C'(1, 2).</p> <p>Another transformation <math>T_2</math> maps the same triangle ABC onto triangle A''B''C'' whose coordinates are A'' (4, 2), B'' (1,4), C'' (2,1).</p> <p><b>a)</b> On the same axes plot triangle ABC, A'B'C' and A''B''C'' (2 marks)</p> <p><b>b)</b> Determine</p> <p><b>i)</b> <math>T_1</math></p> <p><b>ii)</b> <math>T_2</math></p> <p><b>iii)</b> the matrix of T such that <math>TT_1 = T_2</math> (6 marks)</p>	
<p>9.</p>	<p><b>1994 Q23 P1</b></p> <p>A rectangle ABCD with vertices A (2, 0), B (4,0), C (4, 4) and D (2,4) is given a stretch transformation with line <math>x = 2</math> as the invariant and point (4, 0) being mapped onto point</p>	

	<p>(6,0).  The image A'B'C'D' of the rectangle is enlarged with a scale factor of -2, centre origin, followed by a reflection in the line <math>y = 0</math>.  On the grid below plot the images of the rectangle ABCD after the successive transformation. (Grid was provided)</p> <p><b>a)</b> Describe the transformation which map the third image onto the first image (2marks)</p> <p><b>b)</b> Describe the single matrix that will map the matrix on the third image onto the first image (2 marks)</p>	Working space
10.	<p><b>1994 Q7 P2</b>  Determine the matrix of transformation that represents the following transformation:  Reflection in <math>x + y = 0</math>, followed by a positive quarter turn about <math>(0, 0)</math> (2marks)</p>	
11.	<p><b>1995 Q 23 P1</b>  On the grid provided on the opposite page ABCE is a trapezium</p> <p>(a) ABCD is mapped onto A 'B 'C 'D' by a positive quarter turn. Draw the image A'B'C 'D' on the grid. (1 mark)</p> <p>(b) A transformation maps <math>\begin{bmatrix} -2 &amp; -1 \\ 1 &amp; -1 \end{bmatrix}</math>  A 'B'C 'D' onto A "B "C "D"</p> <p>(i) Obtain the coordinates of A "B" C" D " on the grid (2 marks)</p> <p>(ii) Plot the image A" B "C "D" on the grid (1mark)</p>	

	<p>(c) Determine a single matrix that maps <math>A''B''C''D''</math> (4 marks)</p>	
12.	<p><b>1997 Q 23 P1</b>  The figure on the grid shows a triangular shaped object ABC and its image A' B 'C'  (a) (i) Describe fully the transformation that maps ABC and A 'B 'C'  (ii) Find a 2 x 2 matrix that transforms triangle ABC onto triangle A 'B'C'  (b) The matrix <math>P = \begin{bmatrix} 2 &amp; 1 \\ 1 &amp; 1 \end{bmatrix}</math> transforms triangle ABC onto A'' B'' C''  (i) Find the coordinates of A ''B ''C''  (ii) Draw the image A'' B'' C''  (c) Find the area of triangle ABC  (d) Hence find the area of the image A''B''C''</p>	Working space
13.	<p><b>1998 Q 19 P1</b>  A quadrilateral ABCD has vertices A (4, -4), B(2, -4), C(6, -6) and D (4, -2)  a) On the grid provided draw the quadrilateral ABCD.  b) A' B 'C 'D' is the image of ABCD under positive quarter turn about the origin. On the same grid draw the image A'B'C'D'  c) A' B 'C' D' is the image of A' B 'C' D' under the transformation given by the matrix <math>\begin{bmatrix} 1 &amp; -2 \\ 0 &amp; 1 \end{bmatrix}</math>  i) determine the coordinators of A''B ''C ''D''  ii) On the same grid draw the quadrilateral A''B ''C ''D''  d) Determine a single matrix that maps ABCD onto A''B ''C ''D''</p>	

<p>14.</p>	<p><b>1999 Q 23 P2</b>  The transformation R given by the matrix  <math>A = \begin{bmatrix} a &amp; b \\ c &amp; d \end{bmatrix}</math> maps <math>\begin{bmatrix} 17 \\ 0 \end{bmatrix}</math> to <math>\begin{bmatrix} 15 \\ 8 \end{bmatrix} \wedge \begin{bmatrix} 0 \\ 17 \end{bmatrix}</math> to  <math>\begin{bmatrix} -8 \\ 15 \end{bmatrix}</math></p> <p>Determine the matrix A giving a,b,c and d as fractions</p> <p>(a) Given that A represents a rotation through the origin determine the angle of rotation</p> <p>(b) S is a rotation though <math>180^\circ</math> about the point (2, 3). Determine the image of (1,0) under S followed by R.</p>	<p>Working space</p>
<p>15.</p>	<p><b>2000 Q 23 P2</b>  The diagram on the grid provided below shows a trapezium ABCD  On the same grid</p> <p>(a) (i) Draw the image A'B'C'D' of ABCD under a rotation of <math>90^\circ</math> clockwise about the origin .  (ii) Draw the image A''B''C''D'' of A'B'C'D' under a reflection in line <math>y = x</math>. State coordinates of A''B''C''D''</p> <p>(b) A'''B'''C'''D''' is the image of A''B''C''D'' under the reflection in the line <math>x=0</math>.</p> <p>Draw the image A''''B''''C''''D'''' and state its coordinates.</p> <p>(c) Describe a single transformation that maps A''''B''''C''''D'''' onto ABCD.</p>	

16.	<p><b>2001 Q 18 P1</b>  The coordinates of the vertices of rectangle PQRS are P (1,1), Q (6,1), R (6,4) and S(1,4)</p> <p>(a) (i) Find the coordinates of the vertices of its image, P'Q'R'S' under the transformation defined by the matrix <math>\begin{bmatrix} 1 &amp; -2 \\ 0 &amp; 1 \end{bmatrix}</math></p> <p>(i) Draw the object and its image on the grid provided</p> <p>(ii) On the same grid draw the image, P''Q''R''S'' of P'Q'R'S' under the transformation given by <math>\begin{bmatrix} 0 &amp; 1 \\ -1 &amp; 0 \end{bmatrix}</math></p> <p>(b) Find a single matrix which will map P''Q''R''S'' onto PQRS</p>	<p style="text-align: center;">Working space</p>
17.	<p><b>2002 Q 22 P1</b>  A triangle T whose vertices are A (2,3) B(5,3) and C (4,1) is mapped onto triangle T<sup>1</sup> whose vertices are A<sup>1</sup> (-4,3) B<sup>1</sup> (-1,3) and C<sup>1</sup> (x,y) by a transformation</p> $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ <p>Find the: (i) Matrix M of the transformation  (ii) Coordinates of C<sub>1</sub></p> <p>b) Triangle T<sup>2</sup> is the image of triangle T<sup>1</sup> under a reflection in the line y = x.</p> <p>Find a single matrix that maps T and T<sup>2</sup>  (8marks)</p>	
18.	<b>2004 Q 21 P2</b>	

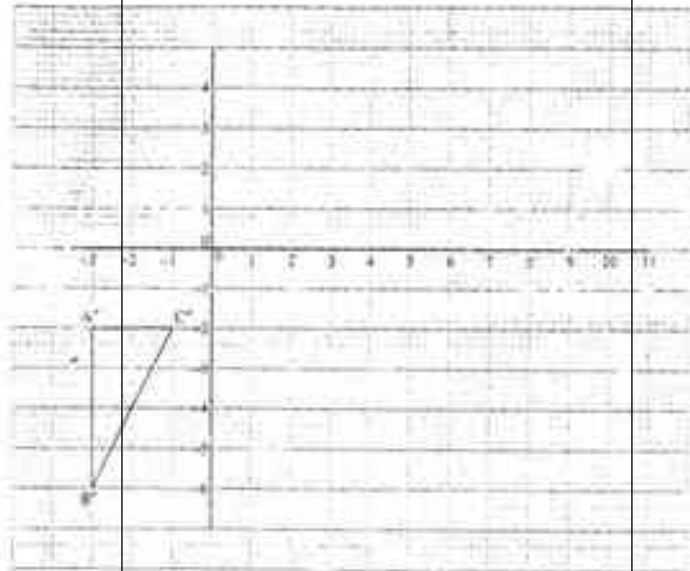


Triangle ABC is the image of triangle PQR under the

transformation  $M = \begin{bmatrix} 2 & 4 \\ 0 & 2 \end{bmatrix}$

Where P,Q and P map onto A, B, and C respectively.

- (a) Given the points P(5, -1), Q(6,-1) and R (4, -0.5), draw the triangle ABC on the grid provided below.



- (b) Triangle ABC in part (a) above is to be enlarged scale factor 2 with centre at (11, -6) to map onto A 'B 'C'. Construct and label triangle A 'B 'C'. on the grid above.

- (c) By construction find the coordinates of the centre and the angle of rotation which can be used to rotate triangle A 'B 'C'. onto triangle A "B "C", shown on the grid above.

Working space

19. **2005 Q 18 P2**

Triangles ABC and A "B "C" are drawn on the Cartesian plane provided.

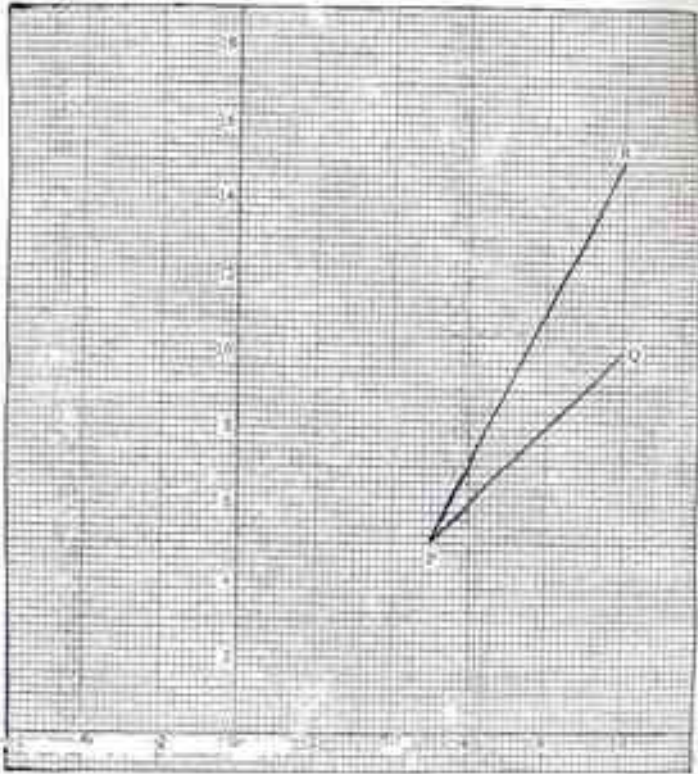
Triangle ABC is mapped onto A "B "C" by two successive transformations

$$R = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{Followed by} \quad \begin{bmatrix} p & 0 \\ -1 & 0 \end{bmatrix}$$

- (a) Find R (4 marks)  
 (b) Using the same scale and axes, draw triangle A "B "C", the image of triangle ABC under transformation R (2 marks)  
 (c) Describe fully, the transformation represented by matrix R (2 marks)

20.	<p><b>2006 Q 19 P2</b>  Triangle ABC is shown on the coordinates plane below  (a) Given that A (-6, 5) is mapped onto A' (6,-4) by a shear with y- axis invariant</p> <p>(i) draw triangle A' B' C', the image of triangle ABC under the shear  (ii) Determine the matrix representing this shear (2 marks)</p> <p>(b) Triangle A' B' C' is mapped on to A'' B'' C'' by a transformation defined by the matrix <math>\begin{bmatrix} -1 &amp; 0 \\ 1\frac{1}{2} &amp; -1 \end{bmatrix}</math></p> <p>(i) Draw triangle A'' B'' C''  (ii) Describe fully a single transformation that maps ABC onto A'' B'' C''</p>	Working space
21.	<p><b>2008 Q 10 P2</b>  Points A(2,2)and B(4,3) are mapped onto A'(2,8) and B'(4,15) respectively by a transformation T. Find the matrix of T. (4marks)</p>	

22.	<p><b>2009 Q 9 P2</b>  The area of triangle FGH is <math>21\text{cm}^2</math>. The triangle FGH is transformed using the matrix <math>\begin{bmatrix} 4 &amp; 5 \\ 1 &amp; 2 \end{bmatrix}</math></p> <p>Calculate the area of the image of triangle FGH (2 marks)</p>	Working space
23.	<p><b>2009 Q 20 P2</b>  Triangle PQR shown on the grid has vertices P(5,5) Q(10,10) and R(10,15)</p>	



- (a) Find the coordinates of the points P', Q' and R', the images of P, Q and R respectively under transformation

M whose matrix is  $\begin{bmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{bmatrix}$

(2marks)

- (b) Given that M is a reflection

(i) Draw triangle P' Q' R' and the mirror line of the reflection (2marks)

(ii) Determine the equation of the mirror line of the reflection (1 mark)

- (c) Triangle P''Q''R'' is the image of triangle P' Q' R' under reflection N where N is a reflection in the y-axis

(i) Draw triangle P''Q''R'' (1 mark)

(ii) Determine a 2 x 2 matrix equivalent to the transformation NM (2 marks)

(iii) Describe fully a single transformation that maps triangle PQR onto triangle P''Q''R'' (2 marks)

Working space

24. **2010 Q 10 P2**

The point O, A and B have the coordinates (0,0), (4,0) and (3,2) respectively

Under shear represented by the matrix  $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

triangle OAB maps onto triangle OAB'

(a) Determine in terms of k, the x coordinates of point B' If (2marks)

(b) OAB' is a right angled triangle in which angle OB' A is acute, find two possible values of k. (2 marks)

25.	<p><b>2011 Q 19 P2</b>  The vertices of a rectangle are A(-1,-1) , B(-4,-1) , C( -4,-3) and D( -1,-3).  a) On the grid provided, draw the rectangle and its image A' B' C' D'</p> <p>Under a transformation whose matrix is <math>\begin{bmatrix} 2 &amp; 0 \\ 0 &amp; -2 \end{bmatrix}</math> (4 marks)</p> <p>b) A'' B'' C'' D'' is the image of A' B' C' D' under a transformation matrix, P = <math>\begin{bmatrix} \frac{1}{2} &amp; 1 \\ 1 &amp; \frac{1}{2} \end{bmatrix}</math></p> <p>i) Determine the coordinates of A'' B'' C'' D'' (2 marks)  ii) On the same grid draw the quadrilateral A''B''C'' D'' (1 mark)</p> <p>c) Find the area of ABCD. (3 marks)</p>	
26.	<p><b>2012 Q18 P2</b>  OABC is a parallelogram with vertices O(0,0), A(2,0), B(3,2) and C(1,2).  O'A'B'C' is the image of OABC under the transformation matrix <math>\begin{bmatrix} -2 &amp; 0 \\ 0 &amp; -2 \end{bmatrix}</math></p> <p>(a) (i) Find the coordinates of O'A'B'C' (2 marks)  ii) On the grid provided draw OABC and O'A'B'C' (2 marks)</p> <p>(b) (i) Find O''A''B''C'', the image of O'A'B'C' under the transformation matrix <math>\begin{bmatrix} 1 &amp; 0 \\ 0 &amp; -2 \end{bmatrix}</math> (2 marks)  (ii) On the same grid draw O''A''B''C''. (1 mark)</p> <p>(c) Find the single matrix that maps O''A''B''C'' onto OABC. (3 marks)</p>	