

KCSE MATHEMATICS PAPER 1

SECTION I (50 marks)

*Answer **all** the questions in this section in the spaces provided.*

1 Evaluate $\frac{22 - 14}{6 \times 2} - \frac{4^2 \times 6 - 12}{72 \div 8 \times 3}$. (2 marks)

2. The production of milk, in litres, of 14 cows on a certain day was recorded as follows: 22, 26, 15, 19, 20, 16, 27, 15, 19, 22, 21, 20, 22 and 28. Determine:

(a) the mode; (1 mark)

(b) the median. (2 marks)

3. Use logarithms, correct to 4 decimal places, to evaluate:

$$\sqrt[3]{\frac{1.794 \times 0.038}{1.243}} \quad (4 \text{ marks})$$

4. Simplify the expression:

$$\frac{16m^2 - 9n^2}{4m^2 - mn - 3n^2} \quad (3 \text{ marks})$$

5. A wholesaler sold a radio to a retailer making a profit of 20%. The retailer later sold the radio for Ksh 1560 making a profit of 30%. Calculate the amount of money the wholesaler had paid for the radio. (3 marks)

6. A point P on the line AB shown below is such that $AP = \frac{1}{3} AB$. By construction locate P. (3 marks)



7. Chelimo's clock loses 15 seconds every hour. She sets the correct time on the clock at 0700h on a Monday. Determine the time shown on the clock when the correct time was 1900h on Wednesday the same week. (3 marks)

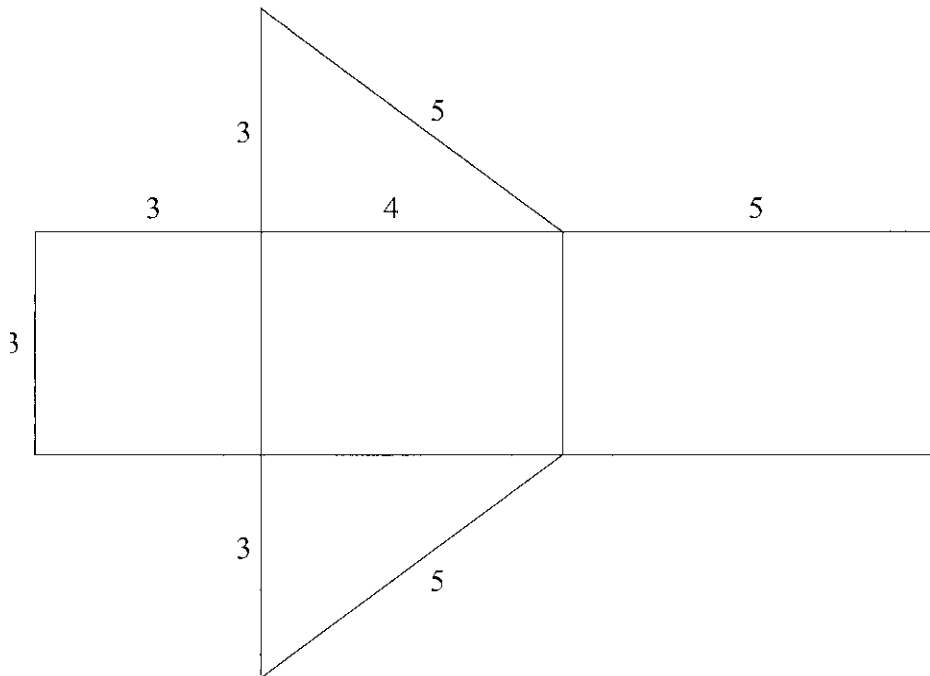
8. Given that $\sin(x + 20)^\circ = -0.7660$, find x , to the nearest degree, for $0^\circ \leq x \leq 360^\circ$. (3 marks)

9. A number m is formed by writing all the prime numbers between 0 and 10 in an ascending order. Another number n is formed by writing all the square numbers between 0 and 10 in a descending order.

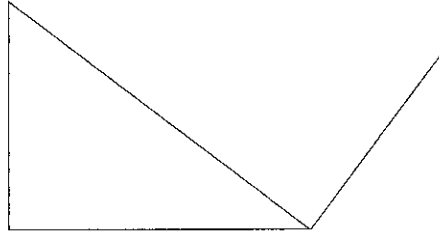
(a) Find $m - n$; (2 marks)

(b) Express $(m - n)$ as a product of its prime factors. (1 mark)

10. The figure below shows a net of a solid. (Measurements are in centimetres).



Below is a part of the sketch of the solid whose net is shown above. Complete the sketch of the solid, showing the hidden edges with broken lines. (3 marks)



11. The interior angles of an octagon are $2x$, $\frac{1}{2}x$, $(x + 40)^\circ$, 110° , 135° , 160° , $(2x + 10)^\circ$ and 185° . Find the value of x . (2 marks)

12. A straight line passes through points $(-2, 1)$ and $(6, 3)$. Find:

(a) the equation of the line in the form $y = mx + c$; (2 marks)

(b) the gradient of a line perpendicular to the line in (a). (1 mark)

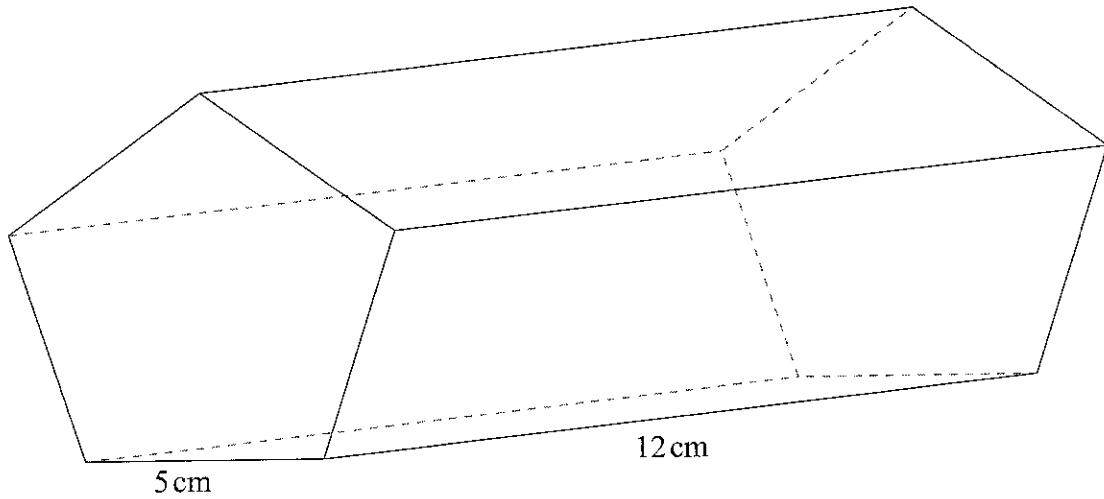
13. A triangle ABC is such that $AB = 5$ cm, $BC = 6$ cm and $AC = 7$ cm.

(a) Calculate the size of angle ACB, correct to 2 decimal places. (2 marks)

(b) A perpendicular drawn from A meets BC at N. Calculate the length AN correct to one decimal place. (2 marks)

14. A cylindrical pipe $2\frac{1}{2}$ metres long has an internal diameter of 21 millimetres and an external diameter of 35 millimetres. The density of the material that makes the pipe is 1.25 g/cm³. Calculate the mass of the pipe in kilograms. (Take $\pi = \frac{22}{7}$). (4 marks)

15. The figure below represents a pentagonal prism of length 12cm. The cross-section is a regular pentagon of side 5 cm.



Calculate the surface area of the prism correct to 4 significant figures. (4 marks)

16 Given the inequalities $x - 5 \leq 3x - 8 < 2x - 3$.

(a) Solve the inequalities; (2 marks)

(b) represent the solution on a number line. (1 mark)

SECTION II (50 marks)

Answer only five questions in this section in the spaces provided.

- 17.** A farmer had 540 bags of maize each having a mass of 112 kg. After drying the maize, the mass decreased in the ratio 15:16.
- (a) Calculate the total mass lost after the maize was dried. (3 marks)
- (b) A trader bought and repacked the dried maize in 90kg bags. He transported the maize in a lorry which could carry a maximum of 120 bags per trip.
- (i) Determine the number of trips the lorry made. (3 marks)
- (ii) The buying price of a 90kg bag of maize was Ksh 1 500. The trader paid Ksh 2 500 per trip to transport the maize to the market. He sold the maize and made a profit of 26%. Calculate the selling price of each bag of the maize. (4 marks)
- 18.** (a) Solve the equation, $\frac{x+3}{24} = \frac{1}{x-2}$. (4 marks)
- (b) The length of a floor of a rectangular hall is 9m more than its width. The area of the floor is 136m^2 .
- (i) Calculate the perimeter of the floor. (4 marks)
- (ii) A rectangular carpet is placed on the floor of the hall leaving an area of 64 m^2 . If the length of the carpet is twice its width, determine the width of the carpet. (12 marks)
- 19.** A trader bought 2 cows and 9 goats for a total of Ksh 98 200. If she had bought 3 cows and 4 goats she would have spent Ksh 2 200 less.
- (a) Form two equations to represent the above information. (2 marks)
- (b) Use matrix method to determine the cost of a cow and that of a goat. (4 marks)

- (c) The trader later sold the animals she had bought making a profit of 30% per cow and 40% per goat.
- (i) Calculate the total amount of money she received. (2 marks)
- (ii) Determine, correct to 4 significant figures, the percentage profit the trader made from the sale of the animals. (2 marks)

20. Two towns, A and B are 80 km apart. Juma started cycling from town A to town B at 10.00 am at an average speed of 40km/h. Mutuku started his journey from townB to town A at 10.30 am and travelled by car at an average speed of 60km/h.

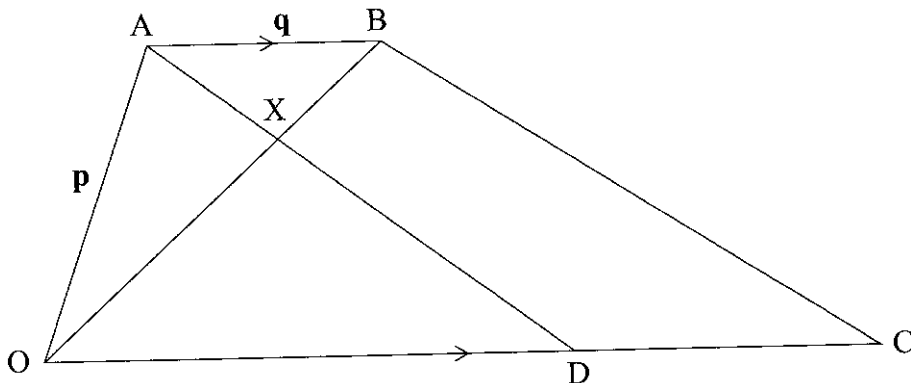
- (a) Calculate:
- (i) the distance from town A when Juma and Mutuku met; (5 marks)
- (ii) the time of the day when the two met. (2 marks)

(b) Kamau started cycling from town A to town B at 10.21am. He met Mutuku at the same time as Juma did. Determine Kamau's average speed. (3 marks)

21. The displacement, s metres, of a moving particle from a point O, after t seconds is given by,
 $s = t^3 - 5t^2 + 3t + 10$.

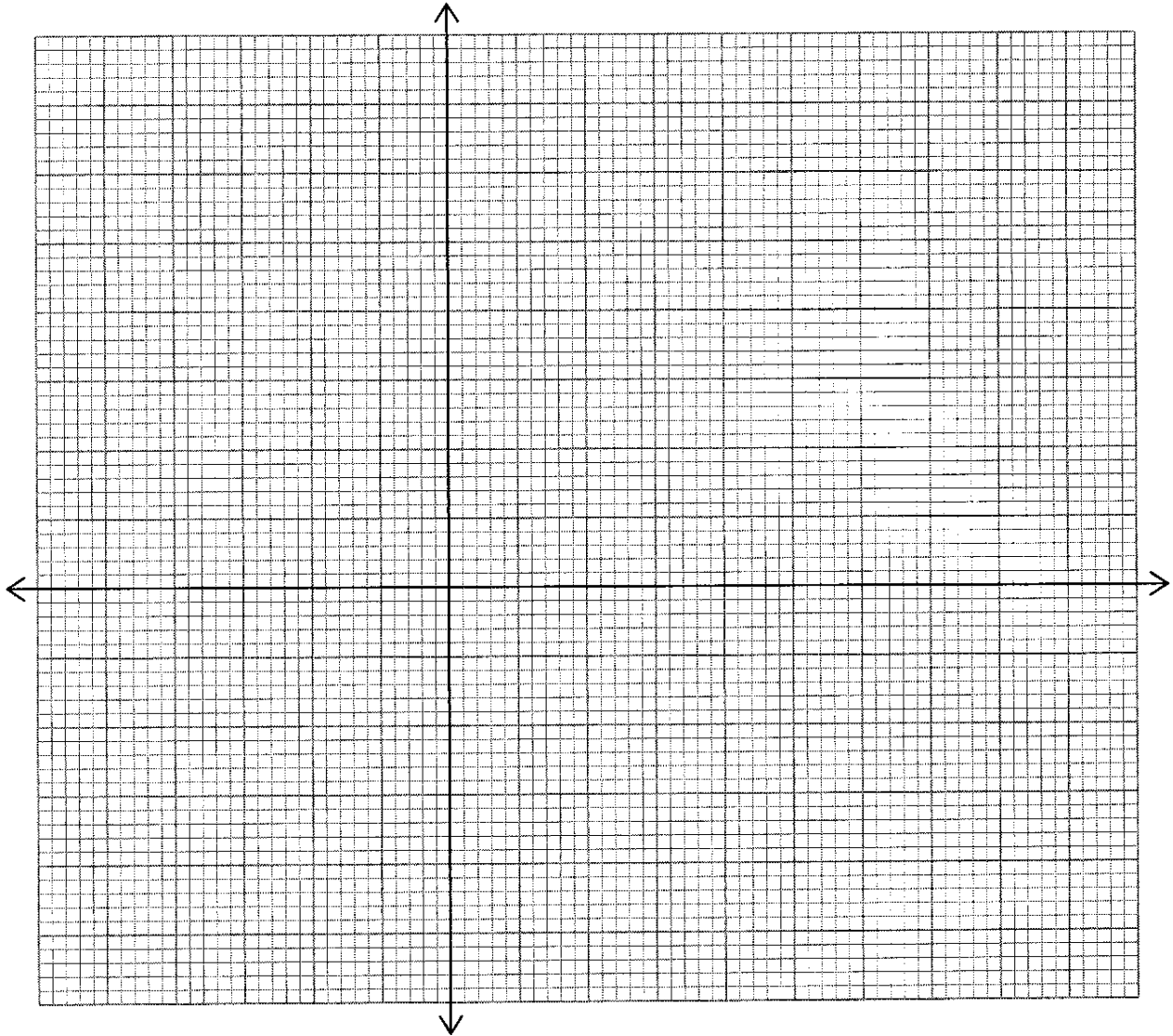
- (a) Find s when $t = 2$. (2 marks)
- (b) Determine:
- (i) the velocity of the particle when $t = 5$ seconds; (3 marks)
- (ii) the value of t when the particle is momentarily at rest. (3 marks)
- (c) Find the time, when the velocity of the particle is maximum. (2 marks)

22. In the figure below, $OABC$ is a trapezium. AB is parallel to OC and $OC = 5AB$. D is a point on OC such that $OD:DC = 3:2$.



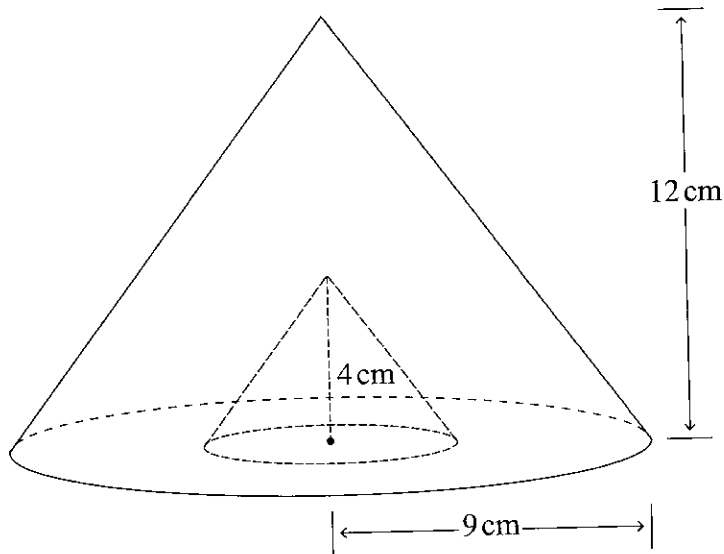
- (a) Given that $\mathbf{OA} = \mathbf{p}$ and $\mathbf{AB} = \mathbf{q}$, express in terms of \mathbf{p} and \mathbf{q} :
- (i) \mathbf{OB} ; (1 mark)
 - (ii) \mathbf{AD} ; (2 marks)
 - (iii) \mathbf{CB} . (2 marks)
- (b) Lines OB and AD intersect at point X such that $\mathbf{AX} = k\mathbf{AD}$ and $\mathbf{OX} = r\mathbf{OB}$, where k and r are scalars. Determine the values of k and r . (5 marks)

23. (a) On the grid provided, draw the square whose vertices are A (6, -2), B (7, -2), C (7, -1) and D (6, -1). (1 mark)



- (b) On the same grid, draw:
- (i) $A'B'C'D'$, the image of $ABCD$, under an enlargement scale factor 3, centre (9, -4); (3 marks)
 - (ii) $A''B''C''D''$, the image of $A'B'C'D'$ under a reflection in the line $x = 0$; (2 marks)
 - (iii) $A'''B'''C'''D'''$, the image of $A''B''C''D''$ under a rotation of $+90^\circ$ about (0,0). (2 marks)
- c) Describe a single transformation that maps $A'B'C'D'$ onto $A'''B'''C'''D'''$. (2 marks)

24. The figure below represents a cone of height 12 cm and base radius of 9 cm from which a similar smaller cone is removed, leaving a conical hole of height 4 cm.



- (a) Calculate:
- (i) the base radius of the conical hole; (2 marks)
 - (ii) the volume, in terms of π , of the smaller cone that was removed. (2 marks)
- (b) (i) Determine the slant height of the original cone. (1 mark)
- (ii) Calculate, in terms of π , the surface area of the remaining solid after the smaller cone is removed. (5 marks)

KCSE 2013 MATHEMATICS PAPER 2

SECTION I (50 marks)

Answer all the questions in this section in the spaces provided

1. The sum of n terms of the sequence; 3, 9, 15, 21, ... is 7500. Determine the value of n .
(3 marks)

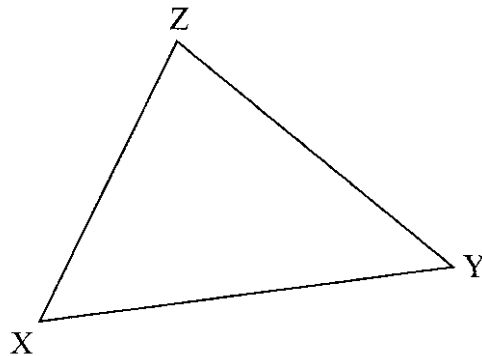
2. A quadratic curve passes through the points (-2, 0) and (1, 0). Find the equation of the curve in the form $y = ax^2 + bx + c$, where a , b and c are constants.
(2 marks)

3. Make d the subject of the fomula,

$$P = \frac{1}{2} mn^2 - \frac{qd^2}{n} \quad (3 \text{ marks})$$

4. Solve the equation
 $2 \log x - \log (x - 2) = 2 \log 3$.
(3 marks)

5. (a) Using a pair of compasses and ruler only, construct an escribed circle to touch side XZ of triangle XYZ drawn below. (3 marks)



- (b) Measure the radius of the circle. (1 mark)

6. The equation of a circle is given by $x^2 + 4x + y^2 - 2y - 4 = 0$. Determine the centre and radius of the circle. (3 marks)

7. (a) Expand $(1 - x)^5$. (1 mark)

- (b) Use the expansion in (a) up to the term in jc^3 to approximate the value of $(0.98)^5$. (2 marks)

8. The position vectors of points F, G and H are \mathbf{f} , \mathbf{g} and \mathbf{h} respectively. Point H divides FG in the ratio 4: -1. Express \mathbf{h} in terms of \mathbf{f} and \mathbf{g} . (2 marks)

9. Two machines, M and N produce 60% and 40% respectively of the total number of items manufactured in a factory. It is observed that 5% of the items produced by machine M are defective while 3% of the items produced by machine N are defective. If an item is selected at random from the factory, find the probability that it is defective. (3 marks)
10. Two taps A and B can each fill an empty tank in 3 hours and 2 hours respectively. A drainage tap R can empty the full tank in 6 hours. Taps A and R are opened for 5 hours then closed.
- (a) Determine the fraction of the tank that is still empty. (2 marks)
- (b) Find how long it would take to fill the remaining fraction of the tank if all the three taps are opened. (2 marks)
11. Simplify the expression $\frac{\sqrt{48}}{\sqrt{5 + \sqrt{3}}}$, leaving the answer in the form $a\sqrt{b} + c$ where a, b and c are integers. (3 marks)
12. A point P moves inside a sector of a circle, centre O, and chord AB such that $2 \text{ cm} < OP \leq 3 \text{ cm}$ and angle APB = 65° . Draw the locus of P. (4 marks)

13. The table below shows income tax rates in a certain year.

Monthly income in Kenya shillings	Tax rate in each shilling
Up to 9 680	10%
from 9 681 to 18800	15%
from 18 801 to 27 920	20%
from 27 921 to 37 040	25%
over 37 040	30%

In that year, a monthly personal tax relief of Ksh 1056 was allowed. Calculate the monthly income tax paid by an employee who earned a monthly salary of Ksh 32 500. (4 marks)

- 14 Solve the equation $6 \cos^2 x + 7 \sin x - 8 = 0$ for $0^\circ \leq x \leq 90^\circ$. (4 marks)

15. The positions of two towns are $(2^\circ\text{S}, 30^\circ\text{E})$ and $(2^\circ\text{S}, 37.4^\circ\text{E})$. Calculate, to the nearest km, the shortest distance between the two towns. (Take the radius of the earth to be 6370 km) (2 marks)

- 16 The vertices of a triangle T are A(1, 2), B(4, 2) and C(3, 4). The vertices of triangle T', the image of T are A'(1/2, 1), B'(2, 1) and C'(3/2, 2).

Determine the transformation matrix $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ that maps T onto T'.

(3 marks)

SECTION II (50 marks)

Answer only five questions from this section in the spaces provided.

17. The Hire Purchase (H.P.) price of a public address system was Ksh 276 000. A deposit of Ksh 60 000 was paid followed by 18 equal monthly instalments. The cash price of the public address system was 10% less than the H.P. price.

(a) Calculate:

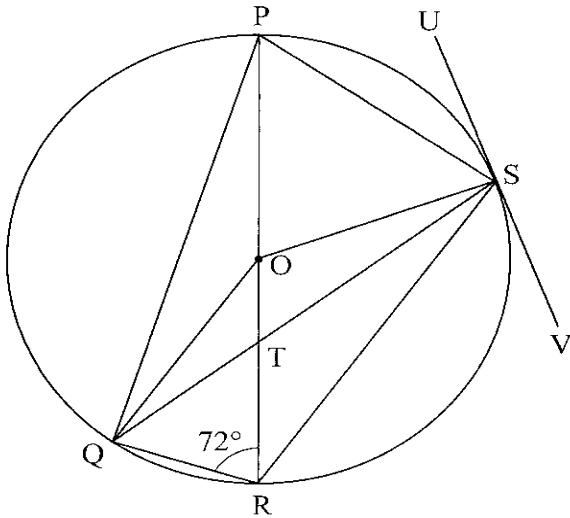
(i) the monthly instalment; (2 marks)

(ii) the cash price. (2 marks)

- b) A customer decided to buy the system in cash and was allowed a 5% discount on the cash price. He took a bank loan to buy the system in cash. The bank charged compound interest on the loan at the rate of 20% p.a. The loan was repaid in 2 years. Calculate the amount repaid to the bank by the end of the second year. (3 marks)

- (c) Express as a percentage of the Hire Purchase price, the difference between the amount repaid to the bank and the Hire Purchase price. (3 marks)

18. In the figure below, PR is a diameter of the circle centre O. Points P, Q, R and S are on the circumference of the circle. Angle PRQ = 72° , QS = QP and line USV is a tangent to the circle at S.



Giving reasons, calculate the size of:

(a) \angle QPR; (2 marks)

(b) \angle PQS; (2 marks)

(c) \angle OQS; (2 marks)

(c) \angle RTS; (2 marks)

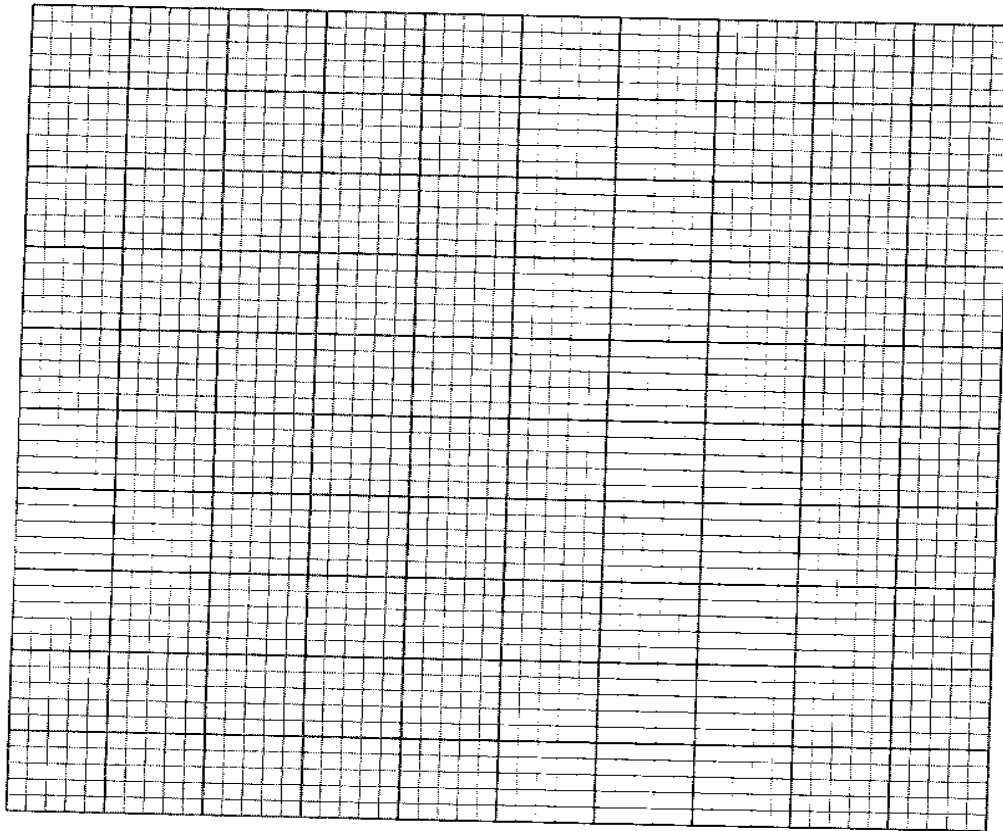
(e) \angle RSV. (2 marks)

- 19 (a) Complete the table below for $y = x^3 + 4x^2 - 5x - 5$. (2 marks)

x	-5	-4	-3	-9	-1	0	1	2
$y = x^3 + 4x^2 - 5x - 5$			19			-5		

- (b) On the grid provided, draw the graph of $y = x^3 + 4x^2 - 5x - 5$ for $-5 \leq x \leq 2$.

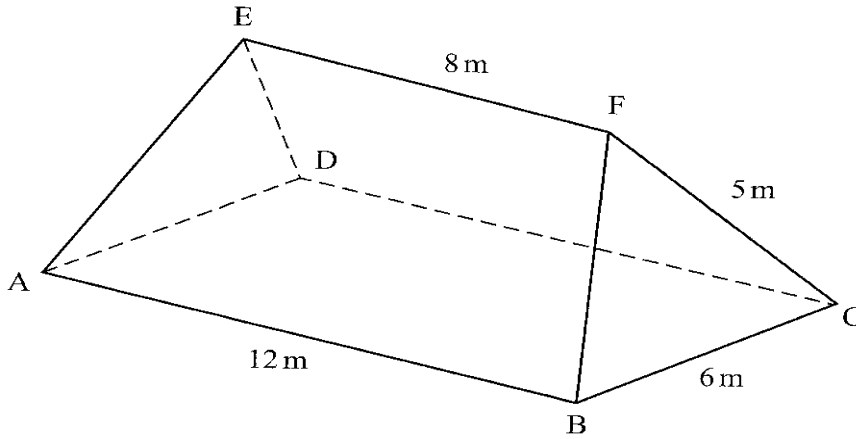
(3 marks)



- c) (i) Use the graph to solve the equation $x^3 + 4x^2 - 5x - 5 = 0$. (2 marks)

- (ii) By drawing a suitable straight line on the graph, solve the equation $x^3 + 4x^2 - 5x - 5 = -4x - 1$. (3 marks)

- 20 The figure ABCDEF below represents a roof of a house. $AB = DC = 12\text{m}$, $BC = AD = 6\text{m}$, $AE = BF = CF = DE = 5\text{m}$ and $EF = 8\text{m}$.

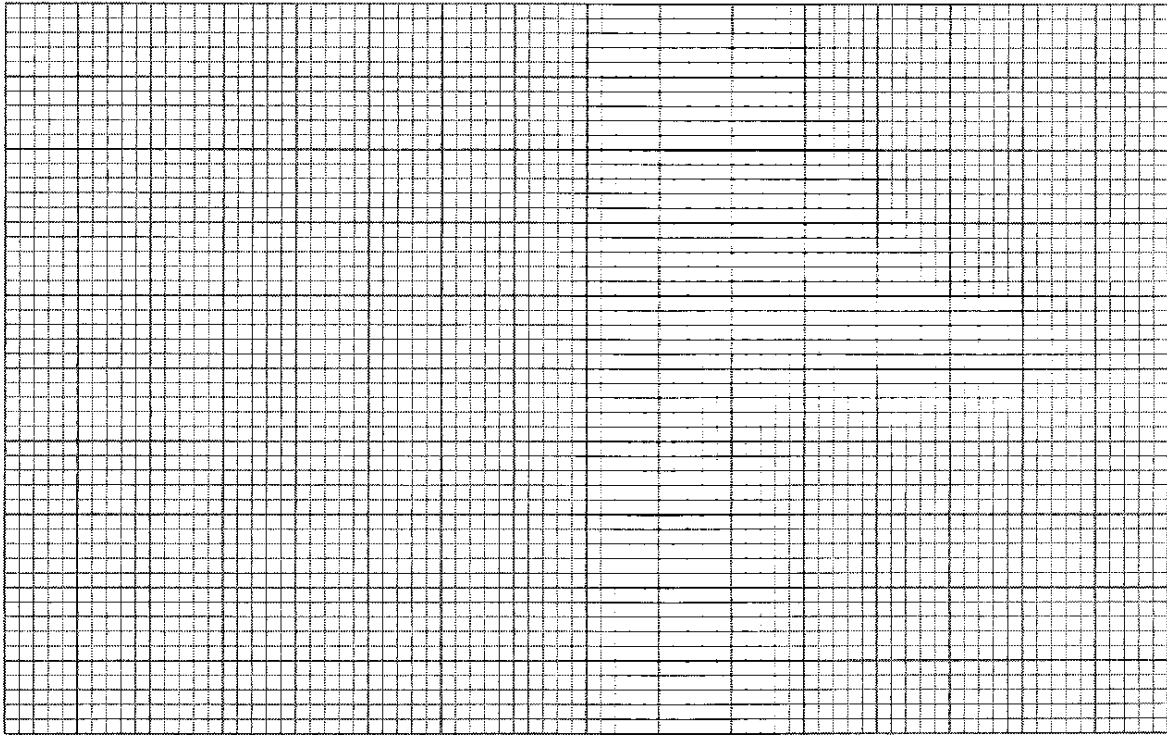


- (a) Calculate, correct to 2 decimal places, the perpendicular distance of EF from the plane ABCD. (3 marks)
- (b) Calculate the angle between:
- the planes ADE and ABCD; (2 marks)
 - the line AE and the plane ABCD, correct to 1 decimal place; (2 marks)
 - the planes ABFE and DCFE, correct to 1 decimal place. (3 marks)

- 21 (a) Complete the table below, giving the values correct to 1 decimal place. (2 marks)

x°	0	40	80	120	160	200	240
$2 \sin (x + 20)^\circ$	0.7		2.0		0.0		- 2.0
$\sqrt{3} \cos x$	1.7	1.3		-0.9		- 1.6	

- (b) On the grid provided, using the same scale and axes, draw the graphs of $y = 2 \sin (x + 20)^\circ$ and $y = \sqrt{3} \cos x$ for $0 \leq x \leq 240^\circ$. (5 marks)



- (c) Use the graphs drawn in (b) above to determine:
 (i) the values of x for which $2 \sin (x + 20) = \sqrt{3} \cos x$; (2 marks)

- ii) the difference in the amplitudes of $y = 2 \sin (x + 20)$ and $y = \sqrt{3} \cos x$. (1 mark)

22. Three quantities R, S and T are such that R varies directly as S and inversely as the square of T.

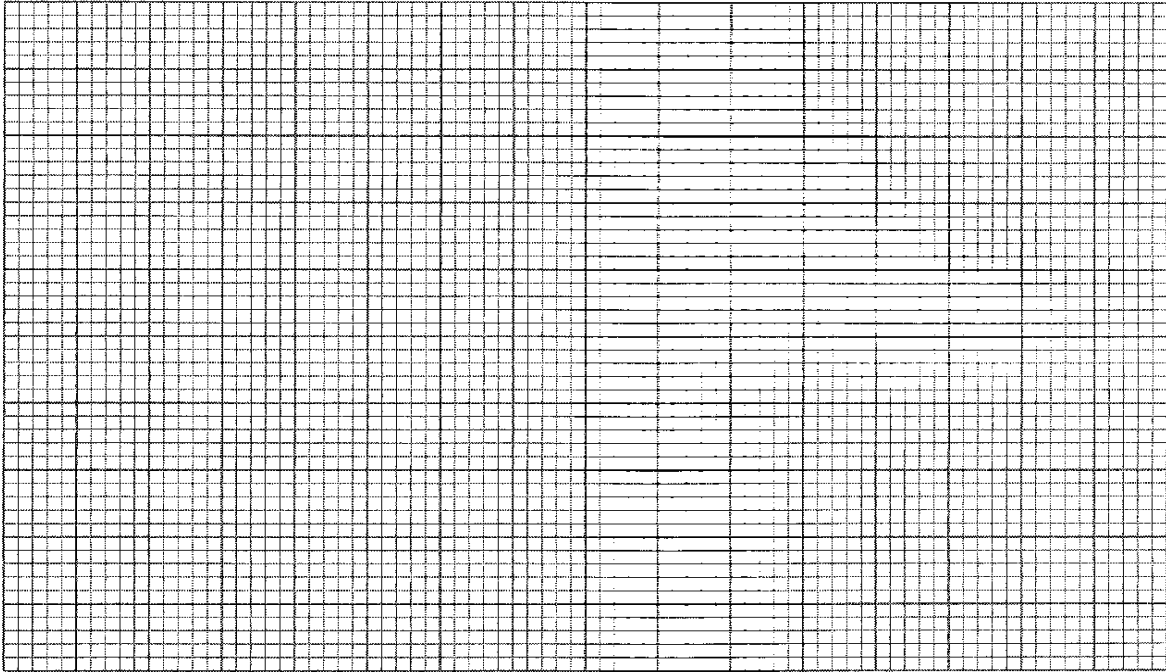
- (a) Given that $R = 480$ when $S = 150$ and $T = 5$, write an equation connecting R, S and T. (4 marks)

- (b) (i) Find the value of R when $S=360$ and $T =1.5$. (2 marks)

- (ii) Find the percentage change in R if S increases by 5% and T decreases by 20%. (4 marks)

23. The equation of a curve is given by $y = 5x - \frac{1}{2}x^2$.

(a) On the grid provided, draw the curve of $y = 5x - \frac{1}{2}x^2$ for $0 \leq x \leq 6$. (3 marks)



(b) By integration, find the area bounded by the curve, the line $x = 6$ and the x -axis. (3 marks)

(c) (i) On the same grid as in (a), draw the line $y = 2x$. (1 mark)

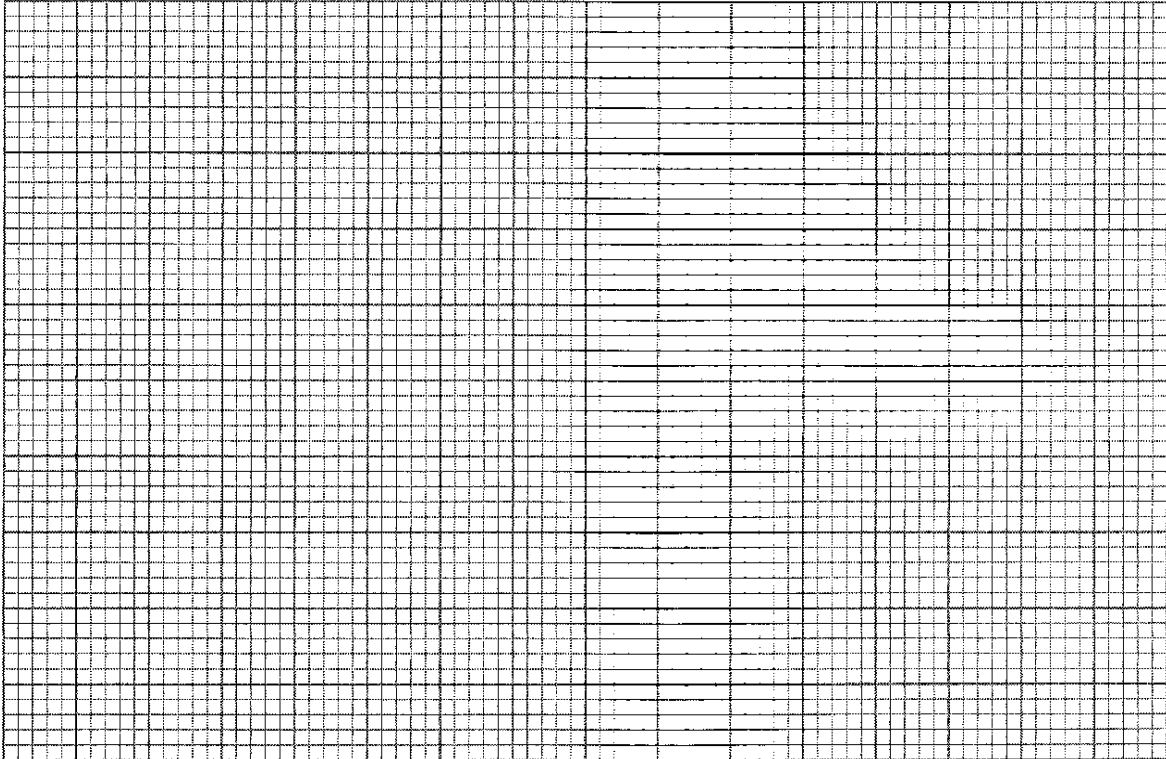
(ii) Determine the area bounded by the curve and the line $y = 2x$. (3 marks)

24. The table below shows marks scored by 42 students in a test.

35	49	69	57	58	75	48
40	46	86	47	81	67	63
56	80	36	62	49	46	26
41	58	68	73	65	59	72
64	70	64	54	74	33	51
73	25	41	61	56	57	28

(a) Starting with the mark of 25 and using equal class intervals of 10, make a frequency distribution table. (2 marks)

(b) On the grid provided, draw the ogive for the data. (4 marks)



9c) Using the graph in (b) above estimate

(i) The median mark; (2 marks)

(ii) The quartile mark (2 marks)