

## GRAPHS OF TRIGONOMETRY EQUATIONS

	KCSE 1989 – 2012 Form 4 Mathematics Answer all the questions	Working space																																										
1.	<p><b>1989 Q23 P2</b>                      Complete the table given below by filling in the blank boxes.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th><math>x^{\circ}</math></th> <th><math>3 \cos x^{\circ}</math></th> <th><math>4 \sin(2x - 10^{\circ})</math></th> </tr> </thead> <tbody> <tr><td><math>0^{\circ}</math></td><td>3.0</td><td>-0.69</td></tr> <tr><td><math>15^{\circ}</math></td><td></td><td>1.37</td></tr> <tr><td><math>30^{\circ}</math></td><td>2.60</td><td></td></tr> <tr><td><math>45^{\circ}</math></td><td></td><td>3.94</td></tr> <tr><td><math>60^{\circ}</math></td><td>1.50</td><td>3.76</td></tr> <tr><td><math>75^{\circ}</math></td><td></td><td></td></tr> <tr><td><math>90^{\circ}</math></td><td>0</td><td>0.69</td></tr> <tr><td><math>105^{\circ}</math></td><td>-0.75</td><td></td></tr> <tr><td><math>120^{\circ}</math></td><td></td><td>-3.06</td></tr> <tr><td><math>135^{\circ}</math></td><td></td><td></td></tr> <tr><td><math>150^{\circ}</math></td><td></td><td>-3.76</td></tr> <tr><td><math>165^{\circ}</math></td><td></td><td></td></tr> <tr><td><math>180^{\circ}</math></td><td>-3.0</td><td>-0.69</td></tr> </tbody> </table> <p style="margin-top: 10px;">Taking 1 cm to represent <math>15^{\circ}</math> on the x-axis and 2cm to represent 1 unit on the y axis, draw the graph of <math>y = 3 \cos x</math> and <math>y = 4 \sin(2x - 10^{\circ})</math> using the same axes on the graph provided.</p> <p style="margin-top: 10px;">Use your graph to find the value of x for which <math>3 \cos x = 4 \sin(2x - 10^{\circ})</math> (8 marks)</p>	$x^{\circ}$	$3 \cos x^{\circ}$	$4 \sin(2x - 10^{\circ})$	$0^{\circ}$	3.0	-0.69	$15^{\circ}$		1.37	$30^{\circ}$	2.60		$45^{\circ}$		3.94	$60^{\circ}$	1.50	3.76	$75^{\circ}$			$90^{\circ}$	0	0.69	$105^{\circ}$	-0.75		$120^{\circ}$		-3.06	$135^{\circ}$			$150^{\circ}$		-3.76	$165^{\circ}$			$180^{\circ}$	-3.0	-0.69	
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	$\cos(x - 15^\circ)$ For $-30^\circ$ $x$ $240^\circ$ (3 marks) Use your graph to find the value of $x$ such that $\cos(x - 15^\circ) = \sin(x - 30^\circ)$ (2marks)	Working space																														
3.	<p><b>1991 Q18 P1</b></p> <p>a) Complete the table given below by filling in the blank boxes. (2marks)</p> <table border="1" data-bbox="229 624 667 983"> <thead> <tr> <th><math>x^\circ</math></th> <th><math>3 \sin x^\circ - 1</math></th> <th><math>\cos x^\circ</math></th> </tr> </thead> <tbody> <tr> <td><math>0^\circ</math></td> <td>-1</td> <td>1</td> </tr> <tr> <td><math>30^\circ</math></td> <td>0.5</td> <td>0.87</td> </tr> <tr> <td><math>60^\circ</math></td> <td></td> <td>0.5</td> </tr> <tr> <td><math>90^\circ</math></td> <td></td> <td>0</td> </tr> <tr> <td><math>120^\circ</math></td> <td></td> <td></td> </tr> <tr> <td><math>150^\circ</math></td> <td></td> <td>-0.87</td> </tr> <tr> <td><math>180^\circ</math></td> <td></td> <td>-1</td> </tr> <tr> <td><math>210^\circ</math></td> <td></td> <td></td> </tr> <tr> <td><math>240^\circ</math></td> <td></td> <td></td> </tr> </tbody> </table> <p>b) On the same axes draw the graph of <math>y = 3 \sin x^\circ - 1</math> and <math>y = \cos x^\circ</math> on the grid below. (grid was provided) (4marks)</p> <p>c) Use your graph to solve the equation <math>3 \sin x^\circ - \cos x^\circ = 1</math> (2marks)</p>	$x^\circ$	$3 \sin x^\circ - 1$	$\cos x^\circ$	$0^\circ$	-1	1	$30^\circ$	0.5	0.87	$60^\circ$		0.5	$90^\circ$		0	$120^\circ$			$150^\circ$		-0.87	$180^\circ$		-1	$210^\circ$			$240^\circ$			
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- a) What is the difference in the values of  $y = 2\sin x$  and  $y = \cos 2x$  at  $x = 67\frac{1}{2}^\circ$ ?  
(2marks)
- b) State the periods of  
 i)  $y = 2\sin x$  (1 mark)  
 ii)  $y = \cos 2x$  (1 mark)

Working space

5. **1995 Q 23 P2**

- (a) Complete the table for the function  $y = 2\sin x$   
(2 marks)

$x^\circ$	$\sin 3x$	$y$
$0^\circ$	0	0
$10^\circ$	0.5000	1.00
$20^\circ$		
$30^\circ$		
$40^\circ$		
$50^\circ$		
$60^\circ$		
$70^\circ$		
$80^\circ$	-0.8666	-1.73
$90^\circ$		
$100^\circ$		
$110^\circ$		
$120^\circ$		

- (b)(i) Using the values in the completed table, draw the graph of  $y = 2\sin 3x$  for  $0^\circ \leq x \leq 120^\circ$  on the grid provided  
 (ii) Hence solve the equation  $2\sin 3x = -1.5$   
(3 marks)

6. **1996 Q 24 P2**

- Complete the table given below for the functions:  
 $y = -3\cos 2x^\circ$  and  $y = 2\sin(\frac{3}{2}x^\circ + 30^\circ)$  for  
 $0^\circ \leq x \leq 180^\circ$  (2 marks)

$x^\circ$	$-3\cos 2x^\circ$	$2\sin(\frac{3}{2}x^\circ + 30^\circ)$
$0^\circ$	-3.00	1.00
$20^\circ$		
$40^\circ$		2.00

$60^\circ$	1.50	1.73
$80^\circ$	2.82	
$100^\circ$	2.82	0.00
$120^\circ$		-1.00
$140^\circ$	-0.52	
$160^\circ$	-2.30	
$180^\circ$		-1.73

a) Using the grid provided, draw the graphs of  $y = -3\cos 2x^\circ$  and  $y = 2\sin (\frac{3}{2}x^\circ + 30^\circ)$  on the same axis. Take 1 cm to represent  $20^\circ$  on the x-axis and 2cm to represent one unit on the y-axis (4 marks)

b) From your graphs, find the roots of  $3\cos 2x^\circ + 2\sin (\frac{3}{2}x^\circ + 30^\circ) = 0$ . (2marks)

Working space

7. **1997 Q 18 P2**  
Complete the table below by filling in the blank spaces

$x^\circ$	$\cos x^\circ$	$2 \cos \frac{1}{2} x^\circ$
$0^\circ$	1.00	2.00
$30^\circ$	0.87	1.93
$60^\circ$	0.50	1.73
$90^\circ$	0	1.41
$120^\circ$	-0.5	1.0
$150^\circ$	-0.87	0.52
$180^\circ$	-1.0	0
$210^\circ$	0.87	0.52
$240^\circ$	-0.5	-1.00
$270^\circ$	0	1.47
$300^\circ$	0.5	1.73
$330^\circ$	0.87	1.93
$360^\circ$	1	-2.00

Using the scale 1 cm to represent  $30^\circ$  on the horizontal axis and 4 cm to represent 1 unit on the vertical axis draw, on the grid provided, the graphs of  $y = \cos x^\circ$  and  $y = 2 \cos \frac{1}{2} x^\circ$  on the same axis.

- (a) Find the period and the amplitude of  $y = 2 \cos \frac{1}{2} x^\circ$
- (b) Describe the transformation that maps the graph of  $y = \cos x^\circ$  on the graph of  $y = 2 \cos \frac{1}{2} x^\circ$

8. **1998 Q 18 P2**

(a) Complete the table below for the value of  $y = 2 \sin x + \cos x$ .

$x^\circ$	$2\sin x$	$\cos x$	$Y$
$0^\circ$	0	1	1
$30^\circ$			
$45^\circ$	1.4	0.7	2.1
$60^\circ$	1.7	0.5	2.2
$90^\circ$	2	0	2
$120^\circ$	1.7	-0.5	1.2
$135^\circ$	1.4	-0.7	0.7
$150^\circ$	1	-0.9	0.1
$180^\circ$	0	-1	-1
$225^\circ$			
$270^\circ$	-2	0	-2
$315^\circ$	-1.4	0.7	-0.7
$360^\circ$	0	1	-1

(b) Using the grid provided draw the graph of  $y = 2 \sin x + \cos x$  for  $0^\circ$ .  
Take 1 cm represent  $30^\circ$  on the x-axis and 2 cm to represent 1 unit on the axis.

(c) Use the graph to find the range of x that satisfy the inequalities  $2 \sin x \cos x > 0.5$

Working space

9. **1999 Q 18 P2**

(a) Complete the table below, giving your values correct to 2 decimal places.

$x$	0	10	20	30	40	50	60	70
<b>Tan x</b>	0							
<b><math>2x + 30^\circ</math></b>	30	50	70	90	110	130	150	170
<b><math>\sin (2x + 30^\circ)</math></b>	0.5			1				

b) On the grid provided, draw the graphs of  $y = \tan x$  and  $y = \sin (2x + 30^\circ)$  for  $0^\circ \leq x < 70^\circ$   
Take scale: 2 cm for  $10^\circ$  on the x-axis  
4 cm for unit on the y-axis  
Use your graph to solve the equation  $\tan x - \sin (2x + 30^\circ) = 0$

10. **2000 Q 24 P2**

(a) Complete the table for the equation  $y = 2\sin(3x + 30^\circ)$  (2 marks)

$x^\circ$	$3x + 30^\circ$	$y = 2\sin(3x + 30^\circ)$
$0^\circ$	$30^\circ$	1
$10^\circ$	$60^\circ$	1.73
$20^\circ$	$90^\circ$	2
$30^\circ$	$120^\circ$	
$40^\circ$	$150^\circ$	
$50^\circ$	$180^\circ$	0
$60^\circ$	$210^\circ$	
$70^\circ$	$240^\circ$	
$80^\circ$	$270^\circ$	-2
$90^\circ$	$300^\circ$	-1.73

(b) Using the grid provided, draw the graph of  $y = 2\sin(3x + 30^\circ)$  for  $0^\circ \leq x \leq 90^\circ$ .  
Take 1cm to represent 40 on the x-axis and 2cm to represent 1 unit on the y axis (3 marks)

(c) Use the graph in (b) to find the range of values of x that satisfy the inequality  $y \leq 1.6$

(3 marks)

Working space

11. 2001 Q 21 P2

a) Complete the table given below in the blank spaces.

$x^\circ$	$3 \cos 2x$	$2 \sin(2x + 30^\circ)$
$0^\circ$	3	1
$15^\circ$	2.598	
$30^\circ$	1.5	2
$45^\circ$	0	2.732
$60^\circ$	1.5	1
$75^\circ$	-3	
$90^\circ$	-2.598	0
$105^\circ$	-1.5	-1
$120^\circ$	0	-1.732
$135^\circ$	2.598	-2
$150^\circ$	3	-2.732

$165^{\circ}$		-2
$180^{\circ}$		1

b) On the grid provided draw, on the same axis, the graph of  $y = 3\cos 2x$  and  $y = \sin(2x + 30^{\circ})$  for  $0^{\circ} \leq x \leq 180^{\circ}$ . Take the scale: 1cm for 150 on the axis and 2cm for 1 unit on the y-axis.

c) Use your graph to estimate the range of value of  $x$  for which  $3 \cos 2x \leq 2\sin (2x+30^{\circ})$ .  
Give your answer to the nearest degree.

12. **2002 Q 23 P2**

a) Complete the table below, giving your values correct to 2 decimal place.

$x^{\circ}$	$\tan \theta^{\circ}$	$\sin \theta^{\circ}$
$0^{\circ}$	0	0
$15^{\circ}$	0.27	0.5
$30^{\circ}$	0.58	
$45^{\circ}$	1	1
$60^{\circ}$	1.73	0.87
$75^{\circ}$		0.5
$90^{\circ}$	$\infty$	0
$105^{\circ}$	3.73	-0.65
$120^{\circ}$	1.73	
$135^{\circ}$	-1	-1
$150^{\circ}$		0.87
$165^{\circ}$	0.27	-0.5
$180^{\circ}$	0	0

b) Using the grid provided and the table in part (a) draw the graphs of  $Y = \tan \theta$  and  $y = \sin 2\theta$ .

c) Using your graphs, determine the range of values for which  $\tan \theta > \sin 2\theta$  for  $0^{\circ} \leq \theta \leq 90^{\circ}$ .

Working space

13. **2003 Q 23 P2**

a) Complete the table below, giving your values correct to 2 decimal places.

$x^{\circ}$	$\tan \theta^{\circ}$	$\sin \theta^{\circ}$
$0^{\circ}$	1	0.5
$15^{\circ}$	0.77	0.17
$30^{\circ}$	0.87	0.87
$45^{\circ}$	0.71	0.97
$60^{\circ}$	0.15	0.10
$75^{\circ}$	0.24	0.97
$90^{\circ}$	0	0.87

<b>105°</b>	-0.26	0.71
<b>120°</b>	-0.5	0.5
<b>135°</b>	-0.17	-0.26
<b>150°</b>	0.5	0
<b>165°</b>	0.87	-0.26
<b>180°</b>	1	-0.5

- b) Using the grid provided draw, on the same axes, the graph of  $y = \cos 2x$  and  $y = \sin (x + 30^\circ)$  for  $0^\circ < x < 180^\circ$  Take the scale: 1cm for  $15^\circ$  on the x axis 4cm for 1 unit on the y- axis. (4 marks)
- c) Find the periods of the curve  $Y = \text{axis}$  (1 marks)
- d) Using the graphs in part (b) above, estimate the solutions to the equation  $\sin (x + 30^\circ) = \cos 2x$  (4marks)

Working space

14. **2005 Q 21 P2**  
(a) Complete the table below, giving your values correct to 2 decimal places

<b>x°</b>	<b>0</b>	<b>30</b>	<b>60</b>	<b>90</b>	<b>120</b>	<b>150</b>	<b>180</b>
<b>2 sin x°</b>	0	1		2		1	
<b>1 - cos x°</b>			0.5	1			



(b) On the grid provided, using the same scale and axes, draw the graphs of  $y = \sin x^\circ$  and  $y = 1 - \cos x^\circ$   $0 \leq x \leq 180^\circ$   
 Take the scale: 2 cm for  $30^\circ$  on the x- axis  
 2 cm for 1 unit on the y- axis

- (c) Use the graph in (b) above to
- (i) Solve equation  $2 \sin x^\circ + \cos x^\circ = 1$  (1 mark)
- (ii) Determine the range of values x for which  $2 \sin x^\circ > 1 - \cos x^\circ$  (1 mark)

Working space

15. **2007 Q 19 P2**

(a) Given that  $y = 8 \sin 2x - 6 \cos x$ , complete the table below for the missing values of y, correct to 1 decimal place.

$x^\circ$	$y = 8 \sin 2x - 6 \cos x$
$0^\circ$	-6
$15^\circ$	-1.8
$30^\circ$	

$45^{\circ}$	3.8
$60^{\circ}$	3.9
$75^{\circ}$	2.4
$90^{\circ}$	0
$105^{\circ}$	
$120^{\circ}$	-3.9

(b) On the grid provided, below, draw the graph of  $y = 8 \sin 2x - 6 \cos x$  for  $0^{\circ} \leq x \leq 120^{\circ}$

Take the scale 2 cm for  $15^{\circ}$  on the x-axis  
2 cm for 2 units on the y-axis

(4 marks)

(c) Use the graph to estimate

(i) The maximum value of y (1 marks)

(ii) The value of x for which  $4 \sin 2x - 3 \cos x = 1$

(3 marks)

Working space

16. **2008 Q 19 P2**

a) Complete the table below, giving the values correct to 2 decimal places.

$x^\circ$	$\sin 2x$	$3\cos x - 2$
$0^\circ$	0	1
$30^\circ$		0.60
$60^\circ$	0.87	
$90^\circ$		-2
$120^\circ$	-0.87	-3.5
$150^\circ$		
$180^\circ$	0	
$210^\circ$	0.87	-4.60
$240^\circ$	0.87	
$270^\circ$		
$300^\circ$		-0.5
$330^\circ$		
$360^\circ$	0	1

b) On the grid provided, draw the graphs of  $y = \sin 2x$  and  $y = 3\cos x - 2$  for

$0^\circ \leq x \leq 360^\circ$  on the same axes. Use a scale of 1 cm to represent  $30^\circ$  on the x-axis and 2 cm to represent 1 unit on the y-axis.

c) Use the graph in (b) above to solve the equation  $3 \cos x - \sin 2x = 2$ . (2 marks)

d) State the amplitude of  $y = 3\cos x - 2$ . (1 mark)

Working space

17. **2010 Q 17 P2**

(a) Complete the table below, giving the value correct to 2 decimal places.

(2 marks)

$x^\circ$	$\cos x^\circ$	$\sin(x^\circ + \cos x^\circ)$
$0^\circ$	1.00	-1.00
$20^\circ$	0.94	-0.60
$40^\circ$	0.77	
$60^\circ$	0.05	0.37
$80^\circ$		0.81
$100^\circ$	-0.17	
$120^\circ$		1.37
$140^\circ$	0.17	
$160^\circ$		1.28
$180^\circ$	-1.00	1.00

b) On the grid provided and using the same axes draw the graphs of  $y = \cos x^\circ$  and  $y = \sin x^\circ - \cos x^\circ$  for  $0^\circ \leq x \leq 180^\circ$ . Using the scale; 1 cm for  $20^\circ$  on the x-axis and 4cm for 1 unit on the y-axis. (5 marks)

c) Using the graph in part (b);

i) Solve the equation  $\sin x^\circ - \cos x^\circ = 1.2$ ; (1 mark)

ii) Solve the equation  $\cos x^\circ = \frac{1}{2} \sin x^\circ$ ; (1 mark)

iii) Determine the value of  $\cos x^\circ$  in part (c) (ii) above. (1 mark)

Working space

18. **2011 Q 16 P2**  
The table below shows values  $x$  and  $y$  for the function  $y=2\sin 3x^\circ$  in the range  $0^\circ \leq x \leq 150^\circ$

$x^\circ$	$y$
$0^\circ$	1
$15^\circ$	1.4
$30^\circ$	2
$45^\circ$	1.4
$60^\circ$	0
$75^\circ$	-1.4
$90^\circ$	-2
$105^\circ$	-1.4
$120^\circ$	0
$135^\circ$	1.4
$150^\circ$	2

- (a) On the grid provided, draw the graph of  $y= 2\sin 3x$  (2 marks)  
(b) From the graph determine the period. (1 mark)

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