

NAME _____ INDEX NUMBER _____

SCHOOL _____ DATE _____

CALCULUS

DIFFERENTIATION

<i>KCSE 1989 – 2012 Form 4 Mathematics</i> <i>Answer all the questions</i>		<i>Working space</i>
1.	1990 Q15 P1 A farmer has 1200m of wire to fence three sides of a rectangular paddock. The fourth side is a wall. Find the dimension that will give the maximum possible area <p style="text-align: right;">(4marks)</p>	
2.	1990 Q11 P2 The gradient of a curve at any point (x, y) is $3x^2$. Given that the curve passes through the point (-2, 3), find its equation. <p style="text-align: right;">(3 marks)</p>	
3.	1991 Q11 P2 Use differentiation to find the x coordinate of the maximum point for the curve $y = x^3 + 2x^2 - 4x - 8$ <p style="text-align: right;">(5 marks)</p>	

		Working space
4.	<p>1992 Q11 P2 Find the equation of the tangent to the curve $y = 2x^2$ at (2, 8) (4marks)</p>	
5.	<p>1993 Q12 P2 Calculate the gradient of the curve $y = x^2 - 3x - 4$ at a point where $x = -1$ (2marks)</p>	
6.	<p>1993 Q24 P2 A projectile is fired vertically upwards. At anytime t (seconds) its height h(metres) above the ground is given by: $h = 30t - 5t^2$</p> <p>a) How fast is it moving at</p> <p>i) $t = 1$ second?</p> <p>ii) $t = 2$ seconds?</p> <p>b) How far up does it travel</p>	

		Working space
7.	<p>1994 Q11 P1 A rectangular plate has a perimeter of 28cm. Determine the dimensions of the plate that give the maximum area</p> <p style="text-align: right;">(4marks)</p>	
8.	<p>1996 Q 19 P1 The equation of a curve is $y = 3x^2 - 4x + 1$</p> <p>(a) Find the gradient function of the curve and its value when $x = 2$ (2 marks)</p> <p>(b) Determine</p> <p>(i) The equation of the tangent to the curve at the point (2, 5) (2 marks)</p> <p>(ii) The angle which the tangent to the curves at the point (2, 5) makes with the horizontal (1 mark)</p> <p>iii) The equation of the line through the point (2, 5) which is perpendicular to the tangent in (b) (i)</p>	

		Working space
9.	1997 Q 10 P1 The curve $y = ax^3 - 3x^2 - 2x + 1$ has the gradient 7 when $x=1$. Find the value of a	
10.	1999 Q 16 P2 Find the equation of the tangent to the curve $y = (x^2 + 1)(x - 2)$ when $x = 2$	

		Working space
11.	<p>2000 Q 5 P2 The distance from a fixed point of a particular in motion at any time t seconds is given by $S = t^3 - 5t^2 + 2t + 5$ metres</p> <p>Find its: (a) Acceleration after t seconds (b) Velocity when acceleration is Zero (c) Find all the integral value of x which satisfy the inequalities $2(2-x) < 4x - 9 < x + 11$</p>	
12.	<p>2001 Q 11 P2 A curve is given by the equation: $y = 5x^3 - 7x^2 + 3x + 2$ Find the: a) Gradient of the curve at $x = 1$ (2 marks)</p> <p>Equation of the tangent to the curve at the point(1,3) (2 marks)</p>	

		Working space
13.	<p>2001 Q 22 P2 The displacement x metres a particle after seconds given by. $x = t^3 - 2t^2 + 6$, $t > 0$.</p> <p>a) Calculate the velocity of the particle in m/s when $t = 2$ seconds.</p> <p>b) When the velocity of the particle is zero, calculate its:-</p> <p>i) Displacement</p> <p>ii) Acceleration.</p>	
14.	<p>2002 Q 16 P1 Given the curve $y = 2x^3 + \frac{1}{2}x^2 - 4x + 1$. Find the:</p> <p>i) Gradient of curve at $\{1, -\frac{1}{2}\}$</p> <p>ii) Equation of the tangent to the curve at $\{1, -\frac{1}{2}\}$</p>	

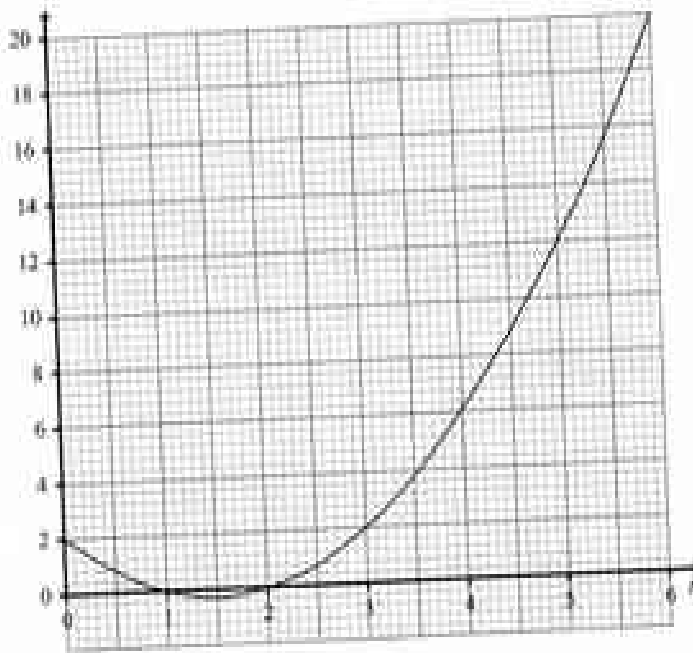
		Working space
15.	<p>2002 Q 24 P2</p> <p>The displacement s metre of a particle moving Along straight line after t seconds is given by.</p> $S = 3t + \frac{3}{2}t^2 - 2t^3$ <p>a) Find its initial acceleration</p> <p>b) Calculate: i) The time when the particle was momentarily at rest. ii) Its displacement by the time it comes to rest momentarily</p> <p>c) Calculate the maximum speed attained.</p>	
16.	<p>2003 Q 8 P2</p> <p>Find the coordinates of the turning point of the curve whose equation is $y = 6 + 2x - 4x^2$ (3 marks)</p>	
17.	<p>2003 Q 21 P2</p> <p>a) i) Find the coordinates of the stationary points on the curve $y = x^3 - 3x + 2$ (2marks)</p> <p>ii) For each stationary point determine whether it is minimum or maximum.</p> <p>b) In the space provided below, sketch the graph of the Function $y = x^3 - 3x + 2$ (2marks)</p>	

		Working space
18.	<p>2004 Q 5 P1</p> <p>The velocity $V \text{ ms}^{-1}$, of a moving body at time t seconds is given by $V = 5t^2 - 12t + 7$. Calculate the acceleration when $t=2$ seconds (3 marks)</p>	
19.	<p>2005 Q 16 P2</p> <p>A stone is thrown vertically upwards from a point O. After t seconds, the stone is S metres from O. Given that $S = 29.4t - 4.9t^2$, find the maximum height reached by the stone (3 marks)</p>	
20.	<p>2005 Q 17 P2</p> <p>A curve is represented by the function $y = \frac{1}{3}x^3 + x^2 - 3x + 2$</p> <p>(a) Find $\frac{dy}{dx}$ (1 mark)</p> <p>(b) Determine the values of y at the turning points of the curve $y = \frac{1}{3}x^3 + x^2 - 3x + 2$ (4 marks)</p>	

		Working space
21.	<p>2006 Q 24 P1</p> <p>A particle moves along straight line such that its displacement S metres from a given point is $S = t^3 - 5t^2 + 4$ where t is time in seconds</p> <p>Find</p> <p>(a) the displacement of particle at $t = 5$ (2 marks)</p> <p>(b) the velocity of the particle when $t = 5$ (3 marks)</p> <p>(c) the values of t when the particle is momentarily at rest (3 marks)</p> <p>The acceleration of the particle when $t = 2$ (2 marks)</p>	
22.	<p>2007 Q 5 P1</p> <p>The gradient of the tangent to the curve $y = ax^3 + bx$ at the point $(1,1)$ is -5. Calculate the values of a and b</p> <p>(4 marks)</p>	
23.	<p>2007 Q 13 P1</p> <p>The sum of two numbers x and y is 40. Write down an expression, in terms of x, for the sum of the squares of the two numbers.</p> <p>Hence determine the minimum value of $x^2 + y^2$</p>	

	(4 marks)	
24.	<p>2008 Q 24 P1 The distance s metres from a fixed point O, covered by a particle after t seconds is given by the equation; $S = t^3 - 6t^2 + 9t + 5$.</p> <p>a) Calculate the gradient to the curve at $t=0.5$ seconds (3 marks)</p> <p>b) Determine the values of s at the maximum and minimum turning points of the curve. (4marks)</p> <p>c) On the space provided, sketch the curve of $s = t^3 - 6t^2 + 9t + 5$. (3 marks)</p>	Working space
25.	<p>2008 Q 15 P2 A particle moves in a straight line from a fixed point. Its velocity Vms⁻¹ after t seconds is given by $V = 9t^2 - 4t + 1$ Calculate the distance travelled by the particle during the third second. (3 marks)</p>	

		Working space
26.	<p>2011 Q 22 P1 The displacement, s metres, of a moving particle after t seconds is given by $s=2t^3 - 5t^2 + 4t + 2$.</p> <p>Determine:</p> <p>a) The velocity of the particle when $t=3$ seconds: (3 marks)</p> <p>b) The value of t when the particle is momentarily at rest; (3 marks)</p> <p>c) The displacement when the particle is momentarily at rest; (2 marks)</p> <p>d) The acceleration of the particle when $t=3$ seconds. (2 marks)</p>	
27.	<p>2012 Q13 P2 The graph below shows the relationship between distance s metres and time t seconds in the interval $0 \leq t \leq 6$.</p>	



Use the graph to determine:

- (a) The average rate of change of distance between $t = 3$ seconds and $t = 6$ seconds; (2 marks)
- (b) The gradient at $t = 3$ seconds. (2 marks)

Working space

28. **2012 Q24 P2**
 The acceleration of a body moving along a straight line is $(4-t) \text{ m/s}^2$ and its velocity is $v \text{ m/s}$ after t seconds,

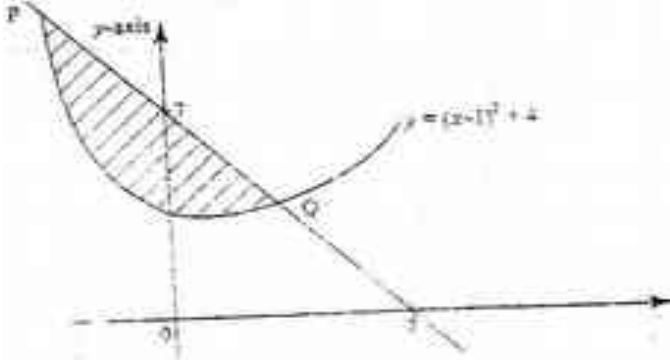
- (a) (i) If the initial velocity of the body is 3 m/s , express the velocity v in terms of t . (3 marks)
- (ii) Find the velocity of the body after 2 seconds. (2 marks)
- (b) Calculate:
- (i) The time taken to attain maximum velocity; (2 marks)
- (ii) The distance covered by the body to attain the maximum velocity (3 marks)

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INTEGRATION KCSE 1989 – 2012 Form 4 Mathematics Answer all the questions		<i>Working space</i>
1.	<p>1989 Q15 P1</p> <p>A particle moves along a straight line PQ. Its velocity v metres per second after t seconds is given by $v = t^2 - 3t + 5$. Its distance from P at the time $t = 1$ is 6metres. Determine its distance from p when $t = 3$.</p> <p style="text-align: right;">(4marks)</p>	
2.	<p>1991 Q14 P1</p> <p>Evaluate</p> $\int_{-1}^3 (2x + 3) dx$	

	(3 marks)	
3.	<p>1992 Q12 P2 The velocity v m/s of a particle moving along a straight line at any time t (sec) is given by $v = 3t - 2$. Its distance x (m) at the time $t = 0$ is equal to 2. Calculate x when $t = 4$</p> <p style="text-align: right;">(4marks)</p>	Working space
4.	<p>1994 Q19 P2 The velocity of a particle moving in a straight line after t seconds given by $v = 6t - t^2 + 4$ m/s. Calculate</p> <p>a) The acceleration of the particle after 2 seconds (2marks)</p> <p>b) The distance covered by the particle between $t = 2$ sec and $t = 5$sec. (3 marks)</p> <p>c) The time when the particle will be momentarily at rest. (3 marks)</p>	

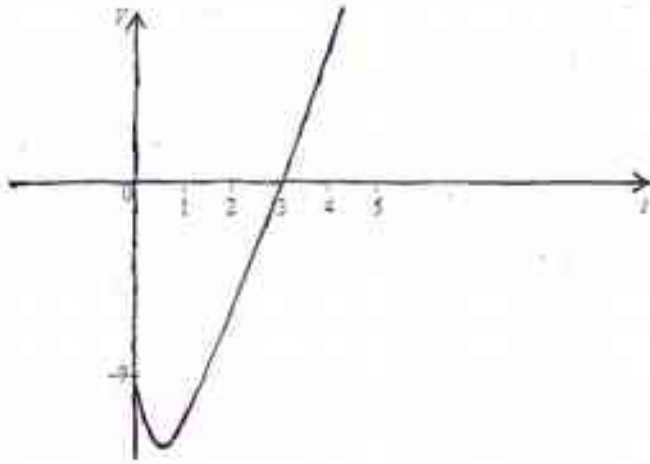
5.	<p>1999 Q 16 P1</p> <p>A particle moves on a straight line. The velocity after t seconds is given by $V = 3t^2 - 6t - 8$. The distance of the particle from the origin after one second is 10 metres.</p> <p>Calculate the distance of the particle from the origin after 2 seconds.</p> <p style="text-align: right;">(4 marks)</p>	Working space
6.	<p>2000 Q 14 P1</p> <p>The acceleration $a \text{ m/s}^2$ of a particle moving in a straight line is given by $a = 18t - 4$, where t is time in seconds. The initial velocity of the particle is 2 m/s</p> <p>a) Find the expression for velocity in terms of t</p> <p>b) Determine the time when the velocity is again 2m/s</p>	

7.	<p>2001 Q 21 P1</p> <p>(a) The gradient function of a curve is given by</p> $\frac{dy}{dx} = 2x^2 - 5$ <p>Find the equation of the curve, given that $y = 3$, when $x = 2$ (4 marks)</p> <p>b) The velocity, v m/s of a moving particle after seconds is given: $v = 2t^3 + t^2 - 1$. Find the distance covered by the particle in the interval $1 \leq t \leq 3$ (4 marks)</p>	Working space
8.	<p>2002 Q 20 P1</p> <p>The diagram below shows a straight line intersecting the curve $y = (x-1)^2 + 4$ at the points P and Q. The line also cuts x-axis at $(7, 0)$ and y axis at $(0, 7)$</p>  <p>a) Find the equation of the straight line in the form $y = mx + c$.</p> <p>b) Find the coordinates of p and Q.</p>	

	<p>c) Calculate the area of the shaded region. (8marks)</p>	
9.	<p>2003 Q 16 P1 The velocity $V\text{ms}^{-1}$ of particle in motion is given by $V = 3t^2 - t + 4$, where t is time in seconds. Calculate the distance travelled by the particle between the time $t=1$ second and $t=5$ seconds.</p> <p style="text-align: right;">(3 marks)</p>	
10.	<p>2004 Q 13 P2 The gradient function of a curve is given $\frac{dy}{dx} = x^2 - 8x + 2$.</p> <p>If the curve passes through the point, $(0, 2)$, find its equation.</p> <p style="text-align: right;">(3 marks)</p>	Working space
11.	<p>2004 Q 22 P2 A particle moves in a straight line. It passes through point O at $t = 0$ with velocity $v = 5\text{m/s}$. The acceleration $a\text{ m/s}^2$ of the particle at time t seconds after passing through O is given by $a = 6t + 4$</p> <p>(a) Express the velocity v of the particle at time t seconds in terms of t (3marks)</p> <p>(b) Calculate (i) The velocity of the particle when $t = 3$ (2 marks) (ii) The distance covered by the particle between $t = 2$ and $t = 4$ (3 marks)</p>	

12.	<p>2005 Q 16 P1 The acceleration, $a \text{ ms}^{-2}$, of a particle is given by $a = 25 - 9t^2$, where t in seconds after the particle passes fixed point O. If the particle passes O, with velocity of 4 ms^{-1}, find</p> <p>(a) An expression of velocity V, in terms of t (2 marks)</p> <p>(b) The velocity of the particle when $t = 2$ seconds (2 marks)</p>	Working space
13.	<p>2005 Q 21 P1 The gradient of a curve at point (x,y) is $4x - 3$. the curve has a minimum value of $-\frac{1}{8}$</p> <p>(a) Find</p> <p>(i) The value of x at the minimum point (1 mark)</p> <p>(ii) The equation of the curve (4 marks)</p> <p>b) P is a point on the curve in part (a) (ii) above. If the gradient of the curve at P is -7, find the coordinates of P (3 marks)</p>	

14.	<p>2006 Q 15 P2</p> <p>A particle moving in a straight line passes through a fixed point O with a velocity of 9m/s. The acceleration of the particle, t seconds after passing through O is given by $a = (10 - 2t) \text{ m/s}^2$.</p> <p>Find the velocity of the particle when t = 3 seconds (3 marks)</p>	
15.	<p>2007 Q 5 P2</p> <p>A particle moves in a straight line through a point P. Its velocity v m/s is given by $v = 2 - t$, where t is time in seconds, after passing P.</p> <p>The distance s of the particle from P when t = 2 is 5 metres. Find the expression for s in terms of t.</p> <p>(3 marks)</p>	
		Working space
16.	<p>2009 Q 16 P2</p> <p>A particle moves in a straight line with a velocity V ms⁻¹. Its velocity after t seconds is given by $V = 3t^2 - 6t - 9$</p> <p>The figure below is a sketch of the velocity-time graph for the particle</p>	



Calculate the distance the particle moves between $t = 1$ and $t = 4$

17. **2010 Q 24 P1**
 A rectangular box open at the top has a square base. The internal side of the base is x cm long and the total internal surface area of the box is 432 cm^2 .
- (a) Express in terms x :
- (i) The internal height h , of the box. (3marks)
 - (ii) The internal volume V , of the box. (1 mark)
- (b) Find:
- i) The value of x for which the volume V is maximum; (4 marks)
 - ii) The maximum internal volume of the box. (2marks)

Working space

18. **2010 Q 11 P2**
 A particle starts from O and moves in a straight line so that its velocity $V \text{ ms}^{-1}$ after time t seconds is given by $V = 3t - t^2$. The distance of the particle from O at time t seconds is s metres.

a) Express s in terms of t and c where c is a constant.
(1 mark)

b) Calculate the time taken before the particle returns to 0.
(3 marks)