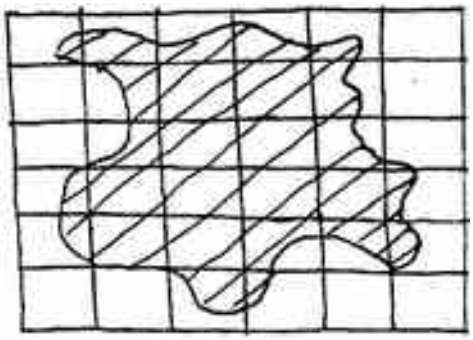
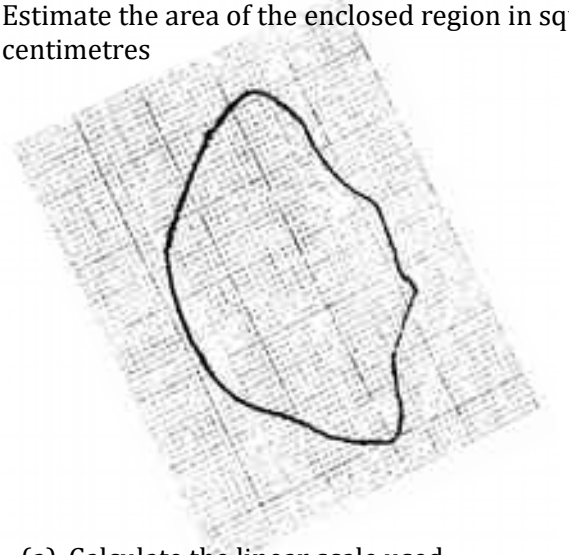
**NAME** _____ **INDEX NUMBER** _____

**SCHOOL** _____ **DATE** _____

# AREA APPROXIMATIONS

| **COUNTING SQUARES TECHNIQUE**<br>*KCSE 1989 – 2012 Form 4 Mathematics*<br>*Answer all the questions* | | Working space |
|---|---|---|
| 1. | **1999 Q 5 P1**<br>The figure below is a map of a forest drawn on a grid of 1 cm squares<br>  <br>i. Estimate the area of the map in square centimeters<br>ii. If the scale of the map is 1: 50,000 estimate the area of the forest in hectares | |
| 2. | **2000 Q 6 P1**<br>The enclosed region shown in the figure below represents a ranch drawn to scale.<br>The actual area of the ranch is 1075 hectares.<br>Estimate the area of the enclosed region in square centimetres<br>  <br>(a) Calculate the linear scale used | |

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| **TRAPEZIODAL** | | Working space |
|---|---|---|
| 1. | **1992 Q24 P2**<br><br>In order to sketch the cross-section of a ditch 175cm wide, the depth of water was measured at intervals of 25cm from one of the banks. The reading of the depths were as follows:<br><br>**i)** Sketch the cross-section of the ditch (3 marks)<br>**ii)** Use trapezoidal rule to estimate the area of the cross-section (5 marks) | |
| 2. | **1993 Q18 P1**<br><br>a) Use the trapezoidal rule to find the area under the curve $y = x^2 + 1$<br><br>From x = 1 to x = 15 using seven strips. (5 marks)<br>b) The cross-section areas in m2 along the length of an 18m wooden log are:<br>5.0, 5.4, 7.0, 8.0, 5.5, 5.8, 6.0<br><br>The cross sectional areas are equally spaced. The first and the last areas represent the ends of the log. Estimate its volume using the trapezoidal rule. (2marks) | |

| Distance (cm) | 0 | 25 | 50 | 75 | 100 | 125 | 150 | 175 |
|---|---|---|---|---|---|---|---|---|
| Depth (cm) | 100 | 115 | 132 | 156 | 167 | 200 | 163 | 153 |

Working space

3. **1996 Q 11 P2**

Complete the table below for the function.

$y= 3x^2 - 8x + 10$

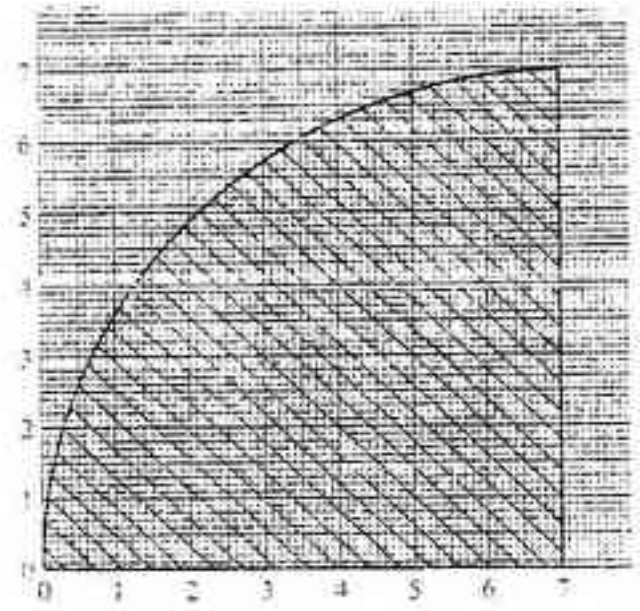
| X | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| y | 10 | 6 | | 70 | | 230 |

Using the values in the table and the trapezoidal rule, estimate the area bounded by the curve $y=3x^2 – 8x + 10$ and the lines y=0,x=0 and x=10

( 3 marks)

4. **1997 Q 8 P2**

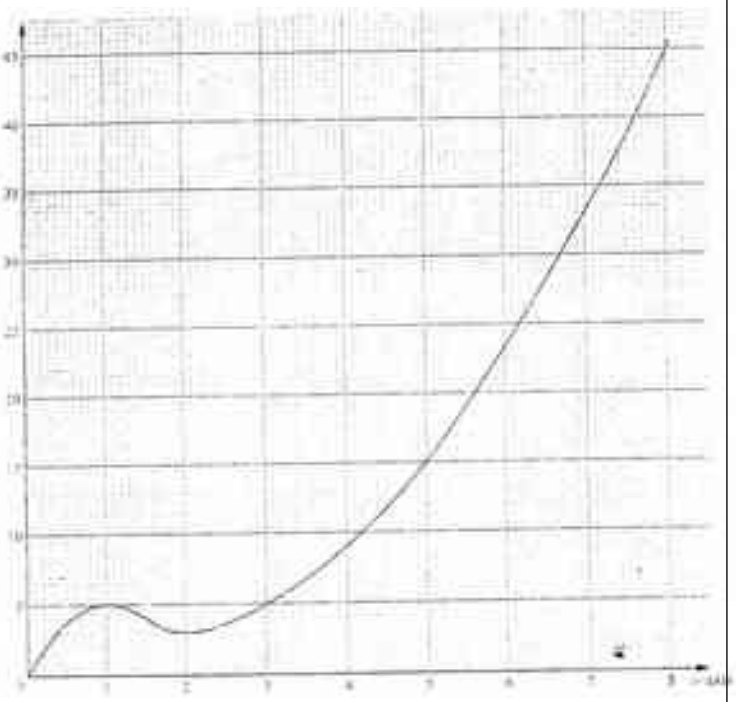
Use the trapezoidal rule with intervals of 1 cm to estimate the area of the shaded region below



Working space

5. **1999 Q 24 P1**

The graph below consists of a non- quadratic part ($0 \le x \le 2$) and a quadrant part ( $2 \le x$ 8) The quadratic part is $y = x^2 - 3x + 5$, $2 \le x \le 8$



(a) Complete the table below

| x | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| y | 3 | | | | | | |

(1 mark)

(b) Use the trapezoidal rule with six strips to estimate the area enclosed by the curve, x = axis and the line x = 2 and x = 8 (3 marks)

(c) Find the exact area of the region given in (b) (3 marks)

(d) If the trapezoidal rule is used to estimate the area under the curve between x = 0 and x = 2, state whether it would give an under- estimate or an over- estimate. Give a reason for your answer (1 mark)

Working space

6. **2001 Q 11 P1**

A particle is projected from the origin. Its speed was recorded as shown in the table below

| Time (sec) | 0 | 5 | 10 | 15 | 20 | 25 | 39 | 35 |
|---|---|---|---|---|---|---|---|---|
| Speed (m/s) | 0 | 2.1 | 5.3 | 5.1 | 6.8 | 6.7 | 4.7 | 2.6 |

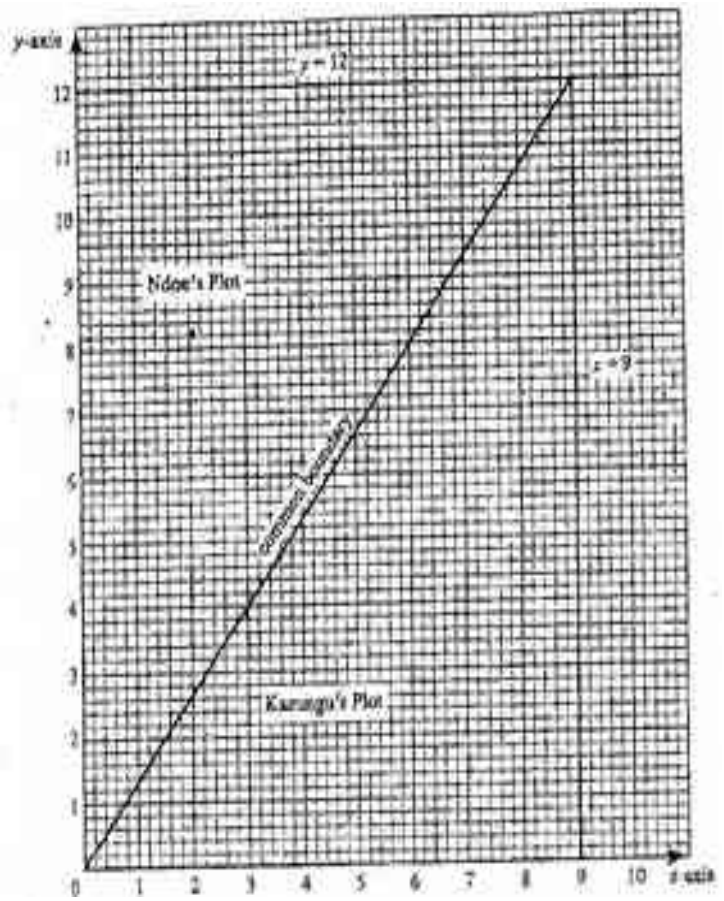
Use the trapezoidal rule to estimate the distance covered by the particle within the 35 seconds

7. **2002 Q 21 P2**

The table below shows the values of x and corresponding values of y for a given curve.

| X | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{5\pi}{12}$ | $\frac{\pi}{2}$ |
|---|---|---|---|---|---|---|---|
| y | 0 | 0.26 | 0.48 | 0.65 | 0.76 | 0.82 | 0.84 |

| | | |
|---|---|---|
| | a) Use the trapezium rule with seven ordinates and the values in the table only to estimate the area enclosed by the curve, x – axis and the line x = $\pi/2$ to four decimal places. (Take $\pi$ = 3.142)<br><br>b) The exact value of the area enclosed by the curve is known to be 0.8940. Find the percentage error made when the trapezium rule is used. Give the answer correct to two decimal places. | Working space |
| 8. | **2006 Q 16 P1**<br>A circle centre O, ha the equation $x^2 + y^2 = 4$. The area of the circle in the first quadrant is divided into 5 vertical strips of width 0.4 cm<br><br>(a) Use the equation of the circle to complete the table below for values of y correct to 2 decimal places<br><br>X \| 0 \| 0.4 \| 0.8 \| 1.2 \| 1.6 \| 2.0<br>Y \| 2.00 \| \| \| 1.60 \| \| 0 (1 mark)<br><br>(b) Use the trapezium rule to estimate the area of the circle (3 marks) | |

| | | |
|---|---|---|
| | | Working space |
| 9. | **2007 Q 24 P1**<br>The diagram on the grid below represents as extract of a survey map showing two adjacent plots belonging to Kazungu and Ndoe.<br>  <br><br>**x** \| **$y_1$** \| **$y_2$** | |

| | | |
|---|---|---|
| **0** | 0 | 0 |
| **1** | 4 | 0.2 |
| **2** | 5.7 | 0.6 |
| **3** | 6.9 | 1.3 |
| **4** | 8 | 2.4 |
| **5** | 9 | 3.7 |
| **6** | 9.8 | 5.3 |
| **7** | 10.6 | 7.3 |
| **8** | 11.3 | 9.5 |
| **9** | 12 | 12 |

On the grid provided above draw and label the boundaries as claimed by Kazungu and Ndoe

(2 marks)

Working space

10. **2011 Q 21 P1**

a) Using the trapezium rule with seven ordinates, estimate the area of the region bounded by the curve $y= -x^2 + 6x + 1$, the lines x=0, y=0 and x=6.

(5 marks)

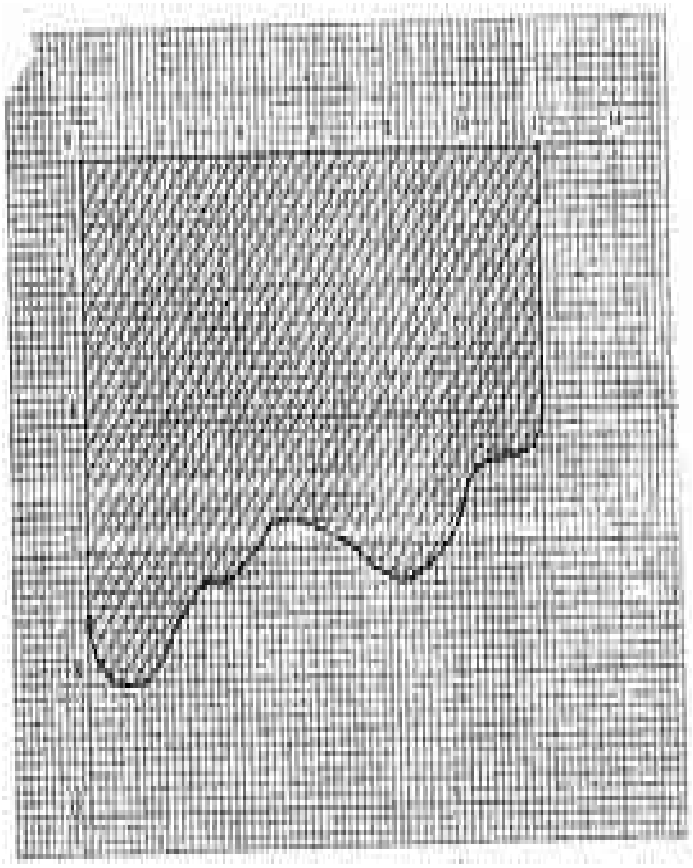
b) Calculate

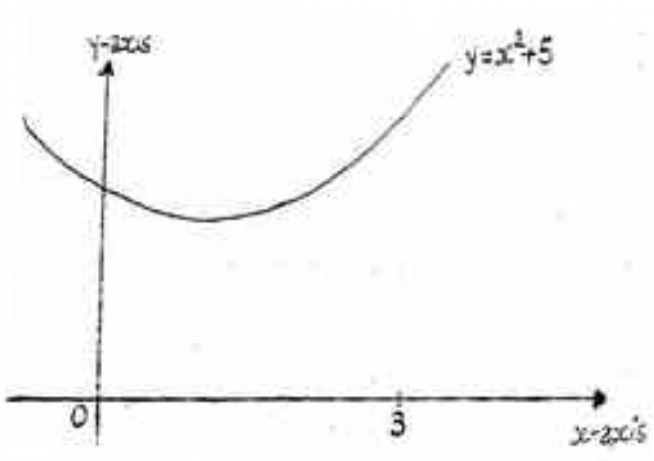
i) the area of the region in a) above by integration:

(3 marks)

ii) the percentage error of the estimated area to the actual area of the region, correct to two decimal places.

(2 marks)

| **MID-ORDINATE** | | Working space |
|---|---|---|
| 1. | **1995 Q 16 P2**<br>The shaded region below represents a forest. The region has been drawn to scale where 1 cm represents 5 km.<br>Use the mid – ordinate rule with six strips to estimate the area of forest in hectares. (4 marks)<br>  | |

| | | |
|---|---|---|
| 2. | **1996 Q 21 P1**<br><br>The table below shows some values of the function<br>$y = x^2 + 2x - 3$<br><br>**x** \| **y**<br>**-6** \| 21<br>**-6.75** \| 18.56<br>**-5.5** \|<br>**-5** \| 14.06<br>**-4.75** \|<br>**-4.5** \| 10.06<br>**4.25** \| 8.25<br>**-4.0** \|<br>**-3.75** \| 5<br>**-3.75** \|<br>**-3.5** \| 2.25<br>**-3.25** \| 1.06<br>**-3** \| 0 | Working space |
| | a) Complete the table<br>b) Using the completed table and the mid- ordinate rule with six ordinates, estimate the area of the region bounded by the $y = x^2 + 2x - 3$ and the line $y = 0$, $x = -6$ and $x = -3$ (3 marks)<br><br>(i) By integration find the actual area of the region in (b) above (2 marks)<br>(ii) Calculate the percentage error arising from the estimate in (b) (2 marks) | |
| 3. | **2003 Q 20 P1**<br>The diagram below is a sketch of the curve $y = x^2 + 5$.<br>  <br><br>a) i) Use the mid –ordinate rule, with six strips to estimate the area enclosed by the curve, the | |

x – axis and the y – axis and line x =3. (4 marks)

ii)Calculate the same area using the integration method. (2marks)

b) Assuming the area calculated in (a) (ii) is exact, calculate the percentage error made when the mid – ordinate rule is used.

4. **2004 Q 11 P1**

The table below shows some values of the function $y = x^2 + 3$

| **x** | **y** |
|---|---|
| **0** | 3 |
| **½** | |
| **1** | |
| **4** | 4 |
| **$1^1/_2$** | $5^1/_4$ |
| **2** | 7 |
| **$2^1/_2$** | |
| **3** | 12 |
| **$3^1/_2$** | $15^1/_4$ |
| **4** | 19 |
| **$4^1/_2$** | |
| **5** | 28 |
| **$5^1/_2$** | |
| **6** | 39 |

Working space

a) Complete the table

b) Use the mid – ordinate rule with six ordinates to estimate the area bounded by $y = x^2+3$, the y – axis, the x – axis and the line x = 6

5. **2005 Q 20 P1**

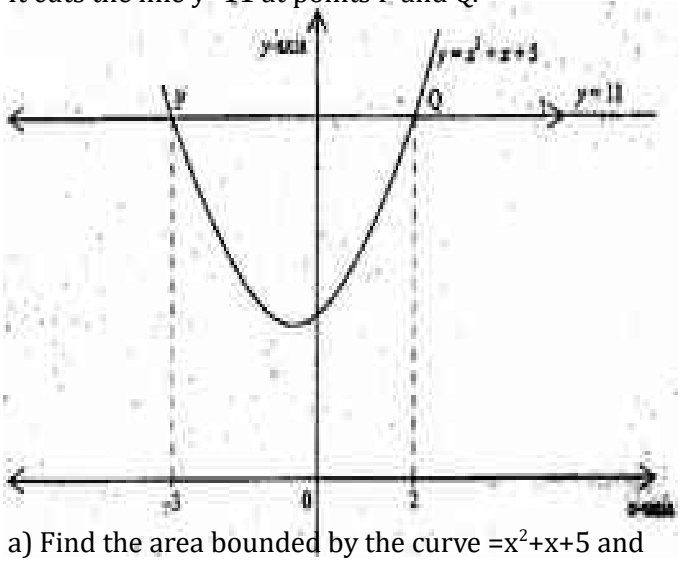
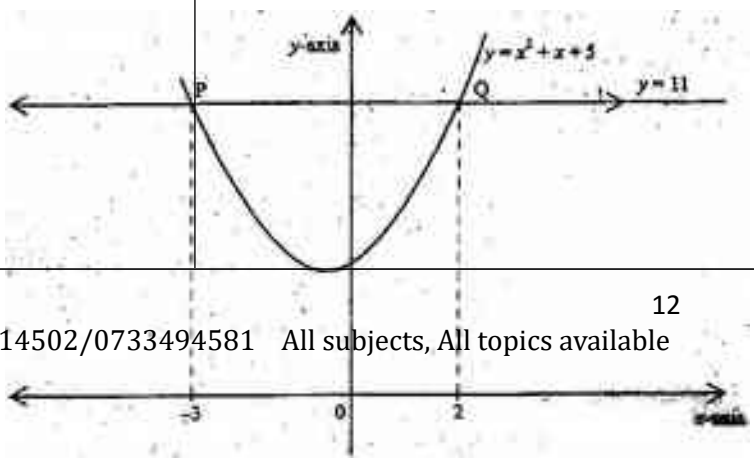
The table below gives some of the values of x for the function $y=½ x^2 + 2x + 1$ in the interval $0 \leq x \leq 6$.

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| y | 1 | 3.5 | 7 | 11.5 | 17 | 23.5 | 31 |

(a) Use the values in the table to draw the graph of the function (2 marks)

(b) (i) Using the graph and the mid – ordinate rule with six (6) strips, estimate the area bounded by the curve, the x- axis, the y-axis and the line = 6

(ii) If the exact area of the region described in (b)

| | | |
|---|---|---|
| | (i) above is $78cm^2$, calculate the percentage error made when the mid – ordinate rule is used.<br>Give the answer correct to two decimal places<br>(2 marks) | |
| 6. | **2008 Q 18 P1**<br>The figure below is a sketch of the curve whose equation is $y=x^2+x+5$.<br>It cuts the line y=11 at points P and Q.<br>  <br><br>a) Find the area bounded by the curve $=x^2+x+5$ and the line y=11 using the trapezium rule with 5 strips. (5 marks)<br>b)Calculate the difference in the area if the mid-ordinate rule with 5 ordinates was used instead of the trapezium rule. (5 marks) | Working Space |
| 7. | **2009 Q 24 P1**<br>(a) On the grid provided, draw a graph of the function $y = ½ x^2 – x + 3$ for $0 \leq x \leq 6$.<br><br>(b) Calculate the mid – ordinates for 5 strips between x = 1 and x=6, and hence use the mid- ordinate rule to approximate the area under the curve between x = 1,x=6 and the x – axis. (3 marks)<br>(c) Assuming that the area determine by integration to be the actual area,<br>calculate the percentage error in using the mid-ordinate rule. (4 marks) |  <br> |

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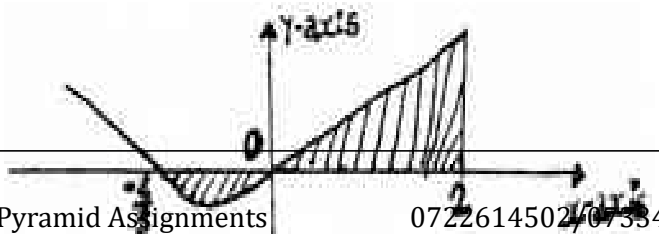
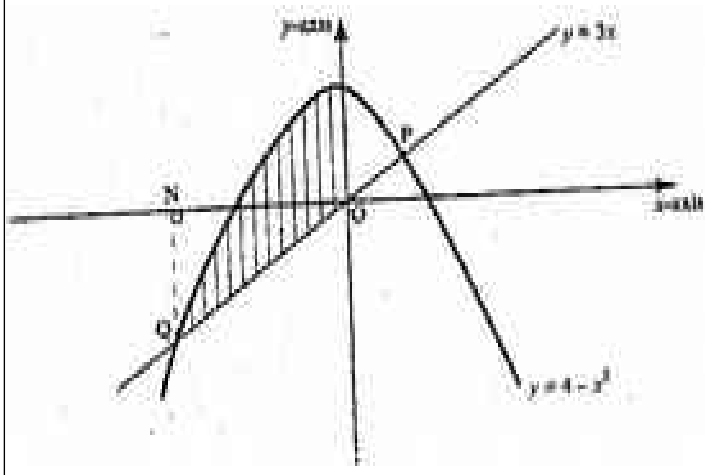
| | INTEGRATION | Working space |
|---|---|---|
| 1. | **1992 Q22 P1**<br><br>**a)** The gradient of the curve $y = ax^2 + bx$ at the origin is equal to 8. Find the values of a and b if the curve has a maximum point at x = 4 (5 marks)<br><br>**b)** Determine the area bounded by the lines x=0, x=6, y=0 and the curve $y=ax^2+bx$, for the values of a and b obtained in part (a) (3 marks) | |

| | | |
|---|---|---|
| | | |
| 2. | **1994 Q6 P2**<br>Determine the area bounded by the curve $y=x^2 - 4$, the x axis and the line x=4<br>(4 marks) | |
| 3 | **3. 1995 Q 7 P1**<br>Find the area enclosed by the curve $y=4x - x^2$, the x- axis and the lines x=1and x=2 (3 marks) | Working Space |
| 4. | **1996 Q 8 P2**<br>Find the area bounded by the curve $y= 2x^3 -5$, the x-axis and the lines x=2 and x=4 | |

| | | |
|---|---|---|
| | | |
| 5. | **1998 Q 20 P2**<br>(a) Find the value of x at which the curve $y= x^2- 2x – 3$ crosses the x- axis (2 marks)<br><br>(b) Find<br>$\int (x^2 – 2x – 3)dx$<br><br>**c)** Find the area bounded by the curve $y = x^2 – 2x – 3$, the axis and the lines $x= 2$ and $x = 4$ | Working Space |
| 6. | **2000 Q 21 P2**<br>The curve of the equation $y = 2x + 3x^2$, has $x = -{}^2/_3$ and $x = 0$ and x intercepts.<br>The area bounded by the axis $x = -{}^2/_3$ and $x = 2$ is shown by the sketch below.<br><br>  | |

| | | |
|---|---|---|
| | Find:<br>(a) $\int(2x + 3x^2)\,dx$<br>(b) The area bounded by the curve x – axis, $x = -^2/_3$ and $x = 2$ | |
| 7 | **2006 Q 24 P2**<br>The diagram below shows a sketch of the line $y = 3x$ and the curve $y = 4 – x^2$ intersecting at points P and Q.<br>  <br>a) Find the coordinates of P and Q<br>(b) Given that QN is perpendicular to the x- axis at N, calculate<br>(i) The area bounded by the curve $y = 4 – x^2$, the x- axis and the line QN (2 marks)<br>(ii) The area of the shaded region that lies below the x- axis<br>(iii) The area of the region enclosed by the curve $y = 4-x^2$, the line $y – 3x$ and the y axis | Working Space |
| 8 | **2007 Q 20 P2**<br>The gradient function of a curve is given by the expression $2x + 1$. If the curve passes through the point ( -4, 6);<br>(a) Find:<br>(i) The equation of the curve (3 marks)<br>(ii) The values of x, at which the curve cuts the x- axis (3 marks)<br>(b) Determine the area enclosed by the curve and the x- axis (4 marks) | |

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